CSE 2001: Introduction to the Theory of Computation
Tutorial 7, Nov 14, 7 pm

Problems:

1. Consider some alphabet Σ. Consider a regular language $A \subseteq \Sigma^*$ and a fixed string $x \in \Sigma^*$. Is the following language regular? Prove your answer.

$$A_x = \{y | xy \in A\}$$

2. Is $\{a^m b^m a^n b^n | m, n \geq 0\}$ regular? Is it context-free?

3. Let $\Sigma = \{a, b, c, d\}$ and let

$$D = \{a^i b^j c^k d^m | i = m, j = k\}$$

Is D context-free? If it is, provide a CFG that generates it, and **argue that your grammar is correct**. If it is not context-free, provide a Pumping Lemma-based proof.

4. Consider the alphabet $\Sigma = \{0, 1\}$. Prove that the following language is non-regular.

$$L = \{0^m 1^n | 2n \leq m \leq 3n, m = 0, 1, 2, \ldots, n = 0, 1, 2, \ldots\}$$

5. Prove that the following language is not context-free.

$$L = \{a^n b a^n b a^n | n \geq 0\}$$

6. Construct a CFG for the following languages:

   (a) All words w such that w starts and ends with the same symbol. Assume $\Sigma = \{0, 1\}$.
   (b) All words w such that $w = a^{3m+1}, m \geq 0$.

7. Prove that the following language is not context free.

$$L = \{a^p | p \text{ is a prime number } \}$$

8. Prove that the following language is not context free.

$$L = \{a^n b^{n^2} | n \geq 0\}$$

9. Show that if L is a CFL, so is L^R.

10. Show that the intersection of a CFL and a regular language is a CFL.

11. Let A, B be regular languages over Σ. Are the following sets regular?

    (a) $\{x | x \in A, x^R \in B\}$.
    (b) $\{x | x \in A, x^R \in B\}$.
    (c) $\{x | x \in A, x^R = x\}$.

12. Is the language $\{a^{pq} | p, q \text{ are primes}\}$ context free?