

Problems:

1. Prove by induction that for every positive integer n the number $16^n + 6^n + 3$ is divisible by 5.
2. Prove by induction that every integer $n \geq 24$ can be expressed in the form $n = 5a + 7b$ for appropriately chosen non-negative integers a, b .
3. In a graph with n nodes, any path of length n edges contains a cycle.
4. Design a DFA for the language that contains only all binary strings of length 3.
5. Design a DFA for the language that contains only binary strings that end in 0110.
6. Design a DFA for the language that contains only binary strings of non-zero length whose bits sum to a multiple of 3.
7. Design a DFA for the language over $\Sigma = \{a, b\}$ that contains all words containing the string $abab$.
8. Design a DFA for the language over $\Sigma = \{a, b\}$ that contains all words **not** ending in aab .
9. Design a DFA for the language over $\Sigma = \{a, b\}$ that contains all words in which the third letter from the right is b .
10. Design a DFA for the language that contains only binary strings in which every odd position is a 1.
11. Design a DFA for the language over $\Sigma = \{a, b, c\}$ that contains all words in which there are an odd number of a 's.
12. Design a DFA for the language that contains only binary strings in which the first and last symbols are different.
13. Consider the alphabet $\Sigma = \{a, b\}$. Design a DFA for the language $L = \{w \mid |w| > 0, \text{ and the difference in the number of } a\text{'s and } b\text{'s is even}\}$.
14. Consider the alphabet $\Sigma = \{a, b\}$. Design a DFA for the language $L = \{w \mid |w| > 0, \text{ and } w \text{ has an even number of } a\text{'s and an odd number of } b\text{'s}\}$.