CSE 2001:
Introduction to Theory of Computation
Fall 2012

Suprakash Datta

datta@cse.yorku.ca

Office: CSEB 3043

Phone: 416-736-2100 ext 77875

Course page: http://www.cs.yorku.ca/course/2001

Next

- Chapter 2:

  - Pushdown Automata

More examples of CFLs

- $L(G) = \{0^n 1^{2n} \mid n = 1, 2, \dots\}$
- $L(G) = \{xx^R \mid x$ is a string over $\{a,b\}\}$
- $L(G) = \{x \mid x$ is a string over $\{1,0\}$ with an equal number of 1's and 0's$\}$

Next: Pushdown automata (PDA)

Add a stack to a Finite Automaton

- Can serve as type of memory or counter
- <u>More powerful</u> than Finite Automata
- Accepts Context-Free Languages (CFLs)
- Unlike FAs, nondeterminism **makes a difference** for PDAs. We will only study non-deterministic PDAs and omit Sec 2.4 (3rd Ed) on DPDAs.

Pushdown Automata

Pushdown automata are for context-free languages what finite automata are for regular languages.
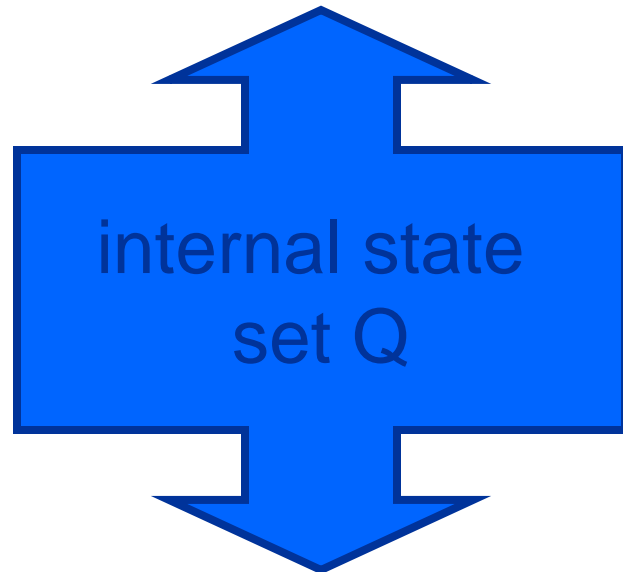
PDAs are *recognizing automata* that have a single stack (= memory):
   Last-In First-Out *pushing* **and** *popping*

Non-deterministic PDAs can make non-deterministic choices (like NFA) to find accepting paths of computation.

Informal Description PDA (1)

input w = 00100100111100101

internal state
set Q

stack

| x |
| y |
| y |
| z |
| x |

The PDA M reads w
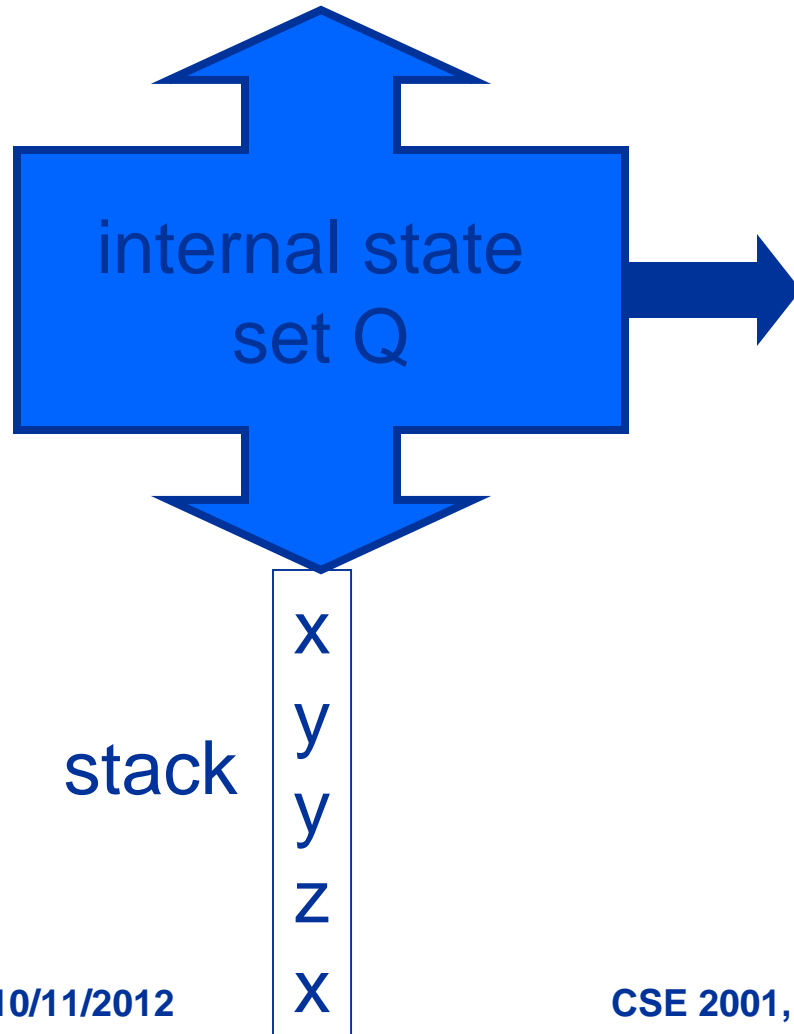and stack element.
<u>Depending on</u>
  - input $w_i \in \Sigma_\varepsilon$,
  - stack $s_j \in \Gamma_\varepsilon$, and
  - state $q_k \in Q$
<u>the PDA M</u>:
  - jumps to a new state,
  - pushes an element Γ_ε
(nondeterministically)

Informal Description PDA (2)

input w = 00100100111100101

internal state
set Q

After the PDA has
read complete input,
M will be in state $\in Q$

If possible to end in
accepting state $\in F \subseteq Q$,
then M accepts w

stack

x
y
y
z
X

Formal Description of a PDA

A Pushdown Automata M is defined by a
six tuple $(Q, \Sigma, \Gamma, \delta, q_0, F)$, with
- Q finite set of states
- Σ finite input alphabet
- Γ finite stack alphabet
- q_0 start state $\in Q$
- F set of accepting states $\subseteq Q$
- δ transition function

$$\delta: Q \times \Sigma_\varepsilon \times \Gamma_\varepsilon \rightarrow \mathcal{P}(Q \times \Gamma_\varepsilon)$$

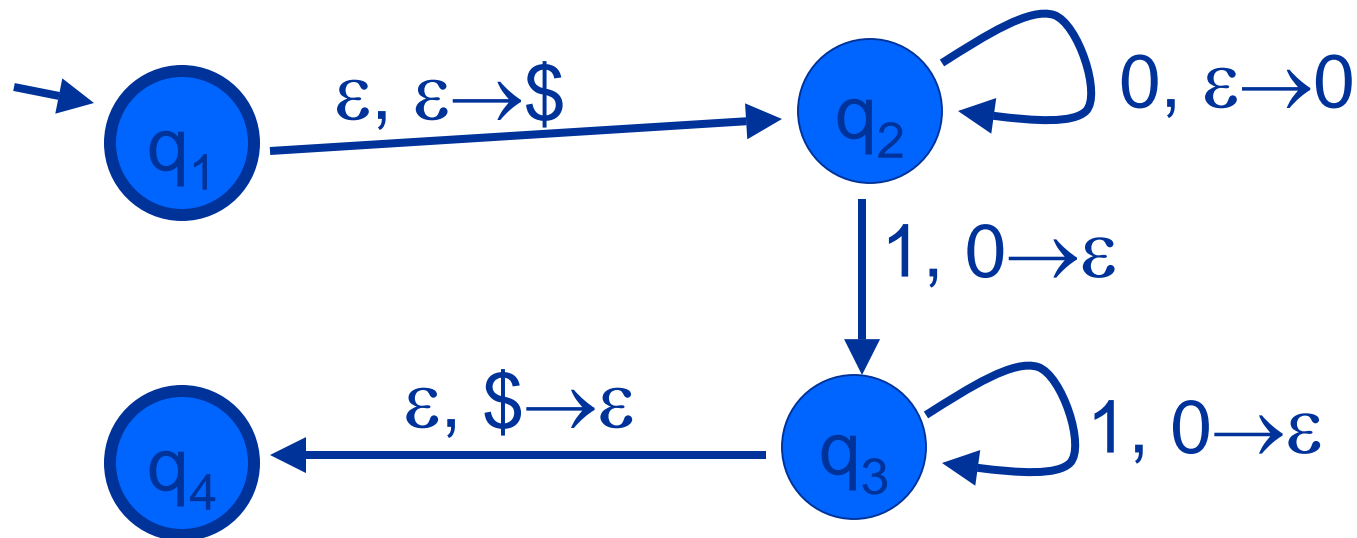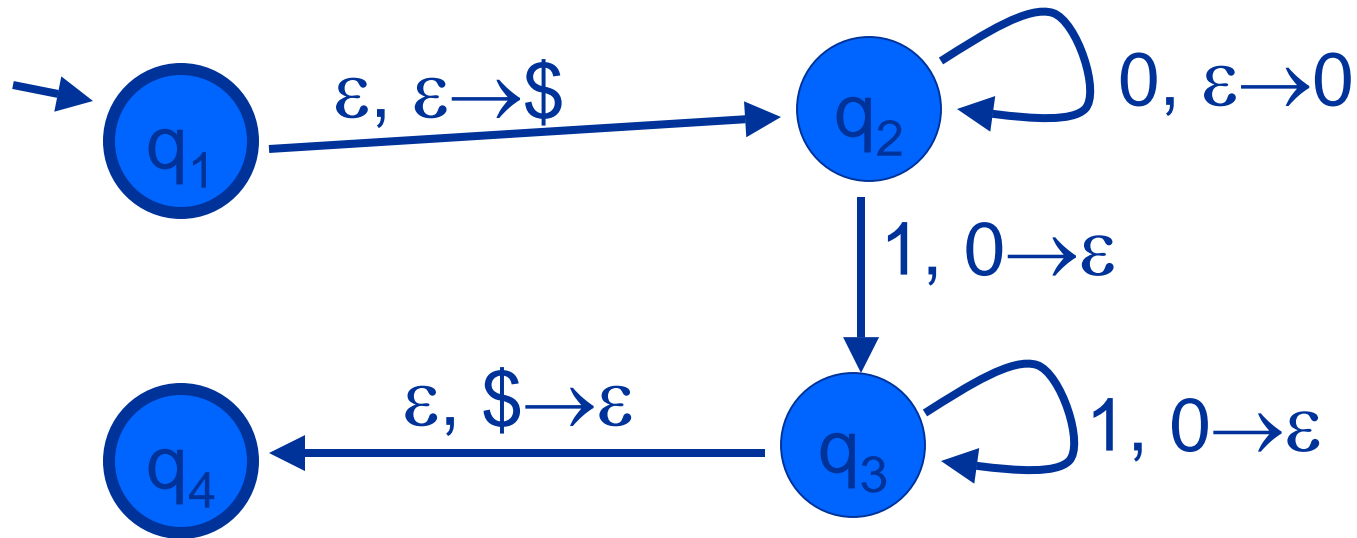# PDA for L = { 0^n1^n | n≥0 }

Example 2.9:
The PDA first pushes " $\$ 0^n$ " on stack.
Then, while reading the 1^n string, the
zeros are popped again.
If, in the end, $\$$ is left on stack, then "accept"

$\varepsilon, \varepsilon \rightarrow \$$

q_1 → q_2

$0, \varepsilon \rightarrow 0$

$1, 0 \rightarrow \varepsilon$

$\varepsilon, \$ \rightarrow \varepsilon$

q_4 ← q_3
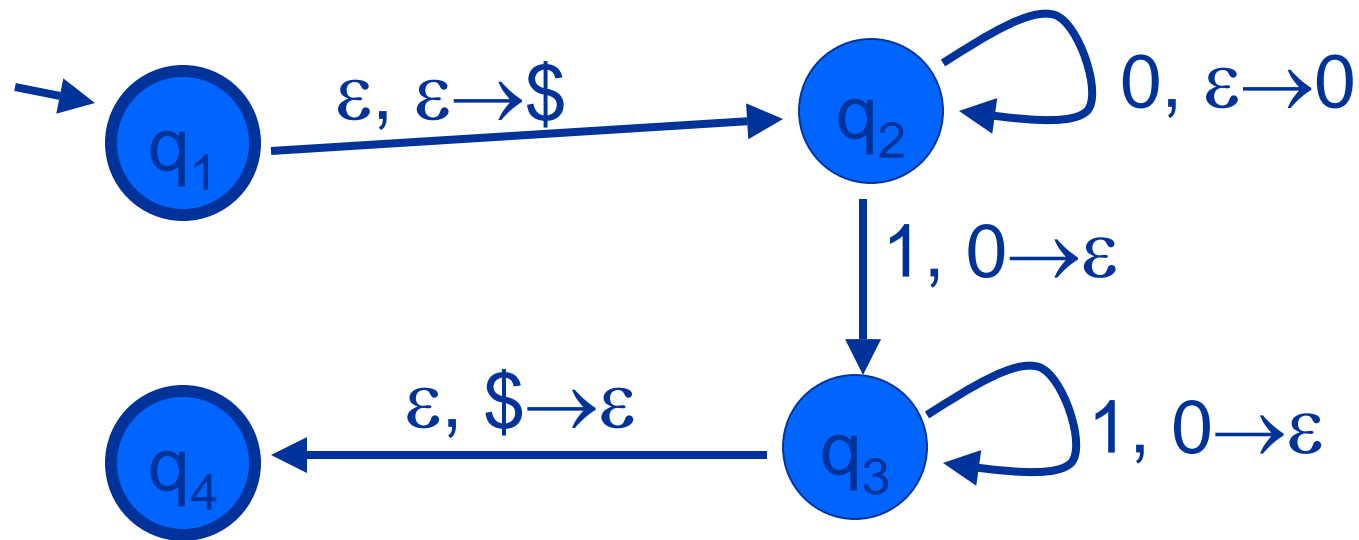
$1, 0 \rightarrow \varepsilon$

Machine Diagram for 0^n1^n

On w = 000111 (state; stack) evolution:

$(q_1; \varepsilon) \rightarrow (q_2; \$) \rightarrow (q_2; 0\$) \rightarrow (q_2; 00\$)$
$\rightarrow (q_2; 000\$) \rightarrow (q_3; 00\$) \rightarrow (q_3; 0\$) \rightarrow (q_3; \$)$
$\rightarrow (q_4; \varepsilon)$ This final q_4 is an accepting state

Machine Diagram for 0^n1^n

On w = 0101 (state; stack) evolution:

$(q_1; \varepsilon) \rightarrow (q_2; \$) \rightarrow (q_2; 0\$) \rightarrow (q_3; \$) \rightarrow (q_4; \varepsilon) \ldots$

But we still have part of input "01".

There is no accepting path.

An important example

- $L = \{a^i b^j a^k \mid i=j \text{ or } i=k \}$
- (Example 2.16, p 115. 3rd ed)

- Try $L = \{ww^R \mid w \text{ ia any binary string} \}$

PDAs and CFL

Theorem 2.20 (2.12 in 2nd Ed):
A language L is context-free if and only if there is a pushdown automata M that recognizes L.

Two step proof:
1) Given a CFG G, construct a PDA M_G
2) Given a PDA M, make a CFG G_M

Converting a CFL to a PDA

- Lemma 2.21 in 3rd Ed

- The PDA should simulate the derivation of a word in the CFG and accept if there is a derivation.

- Need to store intermediate strings of terminals and variables. How?

Idea

- Store only a suffix of the string of terminals and variables derived at the moment starting with the first variable.

- The prefix of terminals up to but not including the first variable is checked against the input.

- A 3 state PDA is enough p 120 3rd Ed.

Converting a PDA to a CFG

- Lemma 2.27 in 3rd Ed
- Design a grammar equivalent to a PDA
- Idea: For each pair of states p,q we have a variable A_{pq} that generates all strings that take the automaton from p to q (empty stack to empty stack).

Some details

Assume

- – Single accept state
- – Stack emptied before accepting
- – Each transition either pops or pushes a symbol

- Can create rules for all the possible cases (p 122 in 3rd Ed)