CSE 2001: Introduction to the theory of computation

Assignment 1 (Released Sept 11, 2012)

Submission deadline: 3:45 pm (in the dropbox) or 4 pm (in class), September 25, 2012

1. The assignment can be handwritten or typed. It MUST be legible.
2. You must do this assignment individually.
3. Submit this assignment only if you have read and understood the policy on academic honesty on the course web page. If you have questions or concerns, please contact the instructor.
4. Use the dropbox near the main office to submit your assignments, or hand them in at the beginning of class (please note the times and day above). No late submissions will be accepted.

Question 1: Sets

Is it possible for two distinct, nonempty sets A, B to satisfy $A \times B \subseteq B \times A$? Give either an example of sets A, B for which this is true or prove that this is not possible.

Question 2: Induction

Use strong induction to prove that every positive integer n can be written as a sum of distinct powers of 2. You will not get credit for this question if your proof is not based on strong induction.
[Hint: for the inductive step, separately consider the case where $k + 1$ is even and where it is odd.]

Question 3

Give a direct proof, a proof by contraposition and a proof by contradiction of the statement: "If n is even, then $n + 4$ is even".

Question 4

In each part below, a recursive definition is given of a subset of $\{a, b\}^*$. Give a simple nonrecursive definition in each case. Assume that each definition includes an implicit last statement: "Nothing is in L unless it can be obtained by the previous statements."

1. $a \in L$; for any $x \in L$, xa, xb are in L.
2. $a \in L$; for any $x \in L$, bx, xb are in L.
3. $a \in L$; for any $x \in L$, ax, xb are in L.
4. $a \in L$; for any $x \in L$, ax, bx, xb are in L.

Question 5

Suppose a language L is defined recursively as:
$\epsilon \in L$; for every x, y in L, $axby$ and $bxay$ are both in L; nothing else is in L. Prove that L is precisely the set of strings in $\{a, b\}^*$ with equal numbers of a'a and b's.