CSE 2001 Final examination
Summer 2007
August 9, 2007
Instructor: S. Datta

Name (LAST, FIRST): ______________________________

Student number: ______________________________

Instructions:

1. If you have not done so, put away all books, papers, cell phones and pagers. Write your name and student number NOW!
2. Check that this examination has 10 pages. There should be 7 questions together worth 90 points, plus 10 bonus points.
3. You have 180 minutes to complete the exam. Use your time judiciously.
4. Show all your work. Partial credit is possible for an answer, but only if you show the intermediate steps in obtaining the answer.
5. If you need to make an assumption to answer a question, please state the assumption clearly.
6. Points will be deducted for **vague and ambiguous** answers.
7. Your answers MUST be LEGIBLE.
8. Feel free not to use the hints supplied.
9. Feel free to invoke the Church-Turing thesis wherever you like, unless the question forbids you from doing so.

Answer the questions in the spaces provided on the question sheets. If you run out of room for an answer, continue on the back of the page.

DO NOT WRITE ANYTHING ON THIS PAGE.

Q	part a	part b	TOTAL
1			
2			
3			
4			
5			
6			
7			

Aggregate score =

1. (a) (9 points) Design a DFA for the language $L = \{w | w \in \{0,1\}^*,\ w$ starts and ends with the same character$\}$. Then convert it to an equivalent CFG.

(b) (6 points) With a small example, show the following. Given a NFA M that accepts a language $L(M)$, you cannot obtain a NFA that accepts the complement of $L(M)$ simply by making accept states of M non-accept and vice versa.

2. (a) (6 points) Let $\sum = \{0, 1, +, =\}$ and

$$ADD = \{x + y = z | x, y, z \text{ are binary integers, and } x \text{ is the sum of } y \text{ and } z\}.$$

Show that ADD is not regular.

(b) (9 points) Let $(x)_r$ be the string obtained from a binary string x by changing 0 to 1 and 1 to 0. For the following language, determine whether it is context-free, and prove your answer.

$$L = \{x(x)_r^R | x \in \{0, 1\}^*\}.$$

3. (a) (7 points) Prove that context-free languages are not closed under intersection and complementation.

(b) (8 points) Determine whether the language $L = \{xwx^R w^R | w, x \in \{0,1\}^*\}$ is context-free and prove your answer.

4. (a) (9 points) Describe a Turing machine that decides $A = \{0^{3^n} | n \geq 0\}$ – the language consisting of all strings of zeroes whose length is a power of 3.

(b) (6 points) Let $\text{INFINITE}_{DFA} = \{\langle A\rangle | A \text{ is a DFA and } L(A) \text{ is an infinite language.}\}$ Is INFINITE_{DFA} decidable? Prove your answer.

5. (a) (8 points) Without using Rice's Theorem (not covered in class) prove the undecidability of the following language:

$$\text{CFL}_{TM} = \{\langle M \rangle | M \text{ is a TM and } L(M) \text{ is a context-free language.}\}$$

(b) (7 points) Without using Rice's Theorem (not covered in class) prove the undecidability of the following language:

$$\text{INFINITE}_{TM} = \{< M > | M \text{ is a TM and } L(M) \text{ is an infinite language.}\}$$

6. (a) (6 points) Show that if a language A is Turing-recognizable and $A \leq_m \overline{A}$, then A is decidable.

(b) (9 points) Recall that an enumerator is a Turing Machine that starts with a blank tape and writes on its tape all strings in its language one after the other, in any order, possibly with repetitions. Show that a language is decidable if and only if some enumerator enumerates the language in lexicographic order.

7. (10 points) [EXTRA CREDIT]

   Consider the problem of determining whether a two-tape Turing Machine ever writes a non-blank symbol on its second tape during the course of its computation on any input string. Formulate this problem as a language and show that it is undecidable.

Use this page if you need extra space. Mark the question number clearly.