MATH/CSE 1019 Discrete Math for Computer Science

Assignment 7

Questions:

1. (3 points) Give a recursive definition of the sequence $a_n = 2^n - 2$, n= 1, 2, 3….

   Solution:
   Basis Step: $a_1 = 0$
   Recursive Step: There are different ways to define a_n. Any of the following will work: (1) $a_n = 2a_{n-1} + 2$ (2) $a_n = a_{n-1} + 2^{n-1}$ (3) $a_n = 3a_{n-1} - 2a_{n-2}$

2. (3 points) Give a recursive definition of the set of perfect squares, i.e. {0, 1, 4, 9, 16 …}.

   Solution:
   Basis Step: $0 \in S$
   Recursive Step: If $a \in S$, then $(\sqrt{a} + 1)^2 \in S$.

3. (3 points) How many **positive** integers between 0 and 9999 inclusive

   (a) are divisible by 3?

   Solution: ⌊9999/3⌋ = 3333

   (b) are not divisible by 9 and 7?

   Solution: 9999 –(⌊ 9999/63⌋)= 9999-158=9841

   (c) are divisible by 9 but not by 7?

   Solution: (⌊ 9999/9 ⌋) – (⌊ 9999/63⌋) = 1111-158=953

4. (3 points) How many 3 digit numbers can you make using the digits 1, 2, 4, 5, and 7 without repetitions?

   Solution: P(5,3)=5*4*3=60

5. (3 points) Suppose there are 8 books on fairy tales, 6 novels and 10 plays. In how many ways can you arrange these so that books on fairy tales are together, novels are together and plays are together?

Solution:

$3!*(8!*6!*10!) = 6.3207*10^{14}$

Note:
First, we consider the books on fairy tales, novels and plays as single objects.
These three categories can be arranged in 3! ways.
Let us fix one of these 6 arrangements.
This may give us a specific order, say, novels $\rightarrow$ fairy tales $\rightarrow$ plays.
Given this order, the books on the same subject can be arranged as follows.
The 8 books on fairy tales can be arranged among themselves in 8! ways.
The 6 novels can be arranged in 6! ways.
The 10 plays can be arranged in 10! ways.
Therefore, for all the 6 possible orders the books can be arranged in 3!*(8!*6!*10!) ways.