MATH/CSE 1019 Discrete Math for Computer Science

Assignment 5

1. (3 points) Let $A = \begin{bmatrix} 0 & 3 & 5 \\ 4 & 1 & 0 \\ 0 & 2 & 7 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & 2 & 2 \\ 1 & 1 & 6 \\ 4 & 0 & 5 \end{bmatrix}$. Find $A^t B$.

$$A^t B = \begin{bmatrix} 0 & 3 & 5 \\ 4 & 1 & 0 \\ 0 & 2 & 7 \end{bmatrix}^t \begin{bmatrix} 3 & 2 & 2 \\ 1 & 1 & 6 \\ 4 & 0 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 4 & 0 \\ 3 & 1 & 2 \\ 5 & 0 & 7 \end{bmatrix} \begin{bmatrix} 3 & 2 & 2 \\ 1 & 1 & 6 \\ 4 & 0 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} 0*3+4*1+0*0 & 0*2+4*1+0*0 & 0*2+4*6+0*5 \\ 3*3+1*1+2*4 & 3*2+1*1+2*0 & 3*2+1*6+2*5 \\ 5*3+0*1+7*4 & 5*2+0*1+7*0 & 5*2+0*6+7*5 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & 4 & 24 \\ 18 & 7 & 22 \\ 43 & 10 & 45 \end{bmatrix}$$

2. (5 points) Use mathematical induction to show that $12^n - 56$ is divisible by 11 for n$\in$N.

   Proof:

   Let the statement to be P(n): $12^n - 56$ is divisible by 11.

   Basis Step: When n=0, $12^0 - 56$ = 1-56 = -55. -55 is divisible by 11. Then P(0) is true.

   Inductive Step: Assume P(k) is true, then $12^k - 56$ is divisible by 11.

   Need to prove P(k+1) is true: $12^{k+1} - 56$ is divisible by 11.

$$12^{k+1} - 56$$
$$= 12^k * 12 - 56$$
$$= 12^k * (11+1) - 56$$
$$= 12^k * 11 + 12^k - 56$$

   From the assumption that P(k) is true, we know that $12^k - 56$ is divisible by 11.

   It is obvious that $12^k * 11$ is divisible by 11 as well.

Then $12^k * 11 + 12^k - 56$ is divisible by 11.

That means $12^{k+1} - 56$ is divisible by 11, and P(k+1) is true.

By mathematical induction, $12^n - 56$ is divisible by 11 for $n \in N$.

Q.E.D.

3. (5 points) Show that $x^3 \log x$ is $O(x^4)$.

Proof:

To show that $x^3 \log x$ is $O(x^4)$, we need to find a pair of witnesses k and C, such that

$\forall x>k, |x^3 \log x| \leqslant C|x^4|$.

When $x>1$, then $x^3 \log x>0$ and $x^4>0$. The inequality becomes $x^3 \log x \leqslant Cx^4$.

$x^3>1$. We can divide both sides of the inequality by x^3. The inequality becomes $\log x \leq Cx$.

∵ $\log x$ is always less than x, when $x>1$.

∴ If we let k=1,C=1, then $\forall x>k, |x^3 \log x| \leq C|x^4|$ is true.

<u>Note: Any k>1 C>1 are be correct witnesses.</u>