MATH/CSE 1019 Discrete Math for Computer Science

Assignment 4

Questions:

1. (7 points) Give a formula for $\sum_{i=0}^{n}\sum_{j=0}^{i}(3j+1)$. Show your steps.

$$\sum_{i=0}^{n}\sum_{j=0}^{i}(3j+1)$$

$$=\sum_{i=0}^{n}(3\sum_{j=0}^{i} j+\sum_{j=0}^{i}1)$$

$$=\sum_{i=0}^{n}(3*\frac{i(i+1)}{2}+(i+1))$$

$$=\sum_{i=0}^{n}(\frac{3}{2}i^2+\frac{5}{2}i+1)$$

$$=\frac{3}{2}\sum_{i=0}^{n}i^2+\frac{5}{2}\sum_{i=0}^{n}i+\sum_{i=0}^{n}1 \qquad (\sum_{i=0}^{n}i^2=\frac{n(n+1)(2n+1)}{6} \text{ is a given formula on page 166.})$$

$$=\frac{3}{2}*\frac{n(n+1)(2n+1)}{6}+\frac{5}{2}*\frac{n(n+1)}{2}+(n+1)$$

$$=\frac{1}{2}(n+1)(n^2+3n+2)$$

2. (7 points) Prove that the Cartesian product of any finite set with any countably infinite set is countably infinite.

Proof:
Given an arbitrary finite set A and an arbitrary countably infinite set B.
Then |A|=k, and $\exists$ bijection $f: Z^+ \to B$, such that $f(n)=b_n$ for $n \in Z^+$

$A \times B = \{< a_i, b_j > | a_i \in A, b_j \in B\}$

If we list the elements of $A \times B$, they are:
$< a_1, b_1 >, < a_1, b_2 >, < a_1, b_3 >, \ldots$
$< a_2, b_1 >, < a_2, b_2 >, < a_2, b_3 >, \ldots$
$\ldots$
$< a_k, b_1 >, < a_k, b_2 >, < a_k, b_3 >, \ldots$

We can count them column by column, i.e. $< a_1, b_1 >, < a_2, b_1 >, \ldots < a_k, b_1 >, < a_1, b_2 >, < a_2, b_2 >, \ldots < a_k, b_2 >, \ldots$

Let $g(n)=<a_r, b_{q+1}>$, where $n=q*k+r$. q is the quotient, r is the remainder, and they are both integers.

Then g(n) is a bijection from Z^+ to $A \times B$.

(1) Prove g(n) is one-to-one.

If $n_1 \neq n_2$, then $< q_1, r_1 > \neq < q_2, r_2 >$, where $n_1 = q_1 * k + r_1$ and $n_2 = q_2 * k + r_2$.

Proof by contrapositive:

Assume $< q_1, r_1 > = < q_2, r_2 >$, *then obviously* $q_1 * k + r_1 = q_2 * k + r_2$.

Then $n_1 = n_2$. −Contrapositive

$< q_1, r_1 > \neq < q_2, r_2 >$ -> $g(n_1) \neq g(n_2)$

Therefore g(n) is one-to-one.

(2) Prove g(n) is onto.

For an arbitrary $<a_i, b_j> \in A \times B$

We can find a pre-image for $<a_i, b_j>$ in Z^+.

Let m= i+k(j-1), then g(m)= $<a_i, b_j>$.

Therefore g(n) is onto.

Q.E.D.