MATH/CSE 1019 Discrete Math for Computer Science

Assignment 3

1. (4 points) Let A, B be sets. Show that $(A-B)\cup(B-A) = (A\cup B)-(A\cap B)$.

   Proof:

   $x\in((A-B)\cup(B-A))$
   $\equiv (x\in(A-B))\vee(x\in(B-A))$ Definition
   $\equiv((x\in A)\wedge(x\notin B))\vee((x\in B)\wedge(x\notin A))$ Definition

   $x\in((A\cup B)-(A\cap B))$
   $\equiv(x\in(A\cup B))\wedge(x\notin(A\cap B))$ Definition
   $\equiv((x\in A)\vee(x\in B))\wedge\neg((x\in A)\wedge(x\in B))$ Definition
   $\equiv((x\in A)\vee(x\in B))\wedge(\neg(x\in A)\vee\neg(x\in B))$ De Morgan's Law
   $\equiv((x\in A)\vee(x\in B))\wedge((x\notin A)\vee(x\notin B))$
   $\equiv\underline{(((x\in A)\vee(x\in B))\wedge(x\notin A))}\vee\underline{(((x\in A)\vee(x\in B))\wedge(x\notin B))}$ Distributive Law
   $\equiv\underline{((x\in A)\wedge(x\notin A))}\vee\underline{((x\in B)\wedge(x\notin A))}\vee\underline{((x\in A)\wedge(x\notin B))}\vee\underline{((x\in B)\wedge(x\notin B))}$ Distributive Law
   $\equiv F\vee((x\in B)\wedge(x\notin A))\vee((x\in A)\wedge(x\notin B))\vee F$ Negation Law
   $\equiv((x\in A)\wedge(x\notin B))\vee((x\in B)\wedge(x\notin A))$ Commutative Law

   $\therefore x\in((A-B)\cup(B-A))\equiv x\in((A\cup B)-(A\cap B))$

   Q.E.D.

2. (6 points) Let $f:S\rightarrow T$. Assume $A\subseteq S$ and $B\subseteq S$.

   (1) (4 points) Show that $f(A\cap B)\subseteq f(A)\cap f(B)$

   Proof:
   For any element y in $f(A\cap B)$, we are going to prove y is also an element of $f(A)\cap f(B)$.
   $\forall y\in f(A\cap B)$
   $\exists x\in(A\cap B)$, such that $f(x)=y$. Definition of a function's Range
   Then $(x\in A)\wedge(x\in B)$. Definition
   Then $(f(x)\in f(A))\wedge(f(x)\in f(B))$.
   $\therefore f(x)\in(f(A)\cap f(B))$
   $\therefore y\in(f(A)\cap f(B))$
   Q.E.D.

(2) (2 points) Disprove $f(A \cap B) = f(A) \cap f(B)$ by giving a counter example.

Proof:
One counter example is: f(x)=|x|, f:R->R. A={-1}, B={1}.
Then $A \cap B = \varnothing$
$f(A \cap B) = f(\varnothing) = \varnothing$
But $f(A) \cap f(B) = \{1\} \cap \{1\} = \{1\}$
$\therefore f(A) \cap f(B) \neq f(A \cap B)$
Q.E.D.

3. (4 points) What is the sum of all integers between 2000 and 3000 which are divisible by 7? Show your steps.

Solution:
2000/7=285.7…
3000/7=428.5…
∴ In [2000,3000], the smallest number that is divisible by 7 is 286*7, and the largest number that is divisible by 7 is 428*7.
∴ All the integers in [2000, 3000] which are divisible by 7 is a sequence:
{286*7, 287*7, 288*7, …, 428*7}
It is an arithmetic progression. The initial term is 286*7. The common difference is 7, and the index ranges from 0 to (428-286)=142.
{286*7, 286*7+7, 286*7+2*7, …, 286*7+142*7}
The sum of the above sequence is

$$\sum_{i=0}^{142} 286*7+7i = \frac{(2*286*7+142*7)(142+1)}{2} = 357357$$