3. x, y can be positive or negative, but at least one should be positive. Therefore, it is the same as $m^3+n^3=200$ or $m^3-n^3=200$ for positive integers m and n.

$1^3=1$, $2^3=8$, $3^3=27$, $4^3=64$, $5^3=125$, $6^3=216$, $7^3=343$, $8^3=512$, $9^3=729$, …

(1) Two positive integers: $m^3<=200$, $n^3<=200$, therefore m,n<=5.

Try all the possible paired sums.
Let m=1, then $m^3=1$. No n can satisfy the equation.
Let m=2, then $m^3=8$. No n can satisfy the equation.
Let m=3, then $m^3=27$. No n can satisfy the equation.
Let m=4, then $m^3=64$. No n can satisfy the equation.

(2) One positive integer and one negative integer:

How to determine the range?
Look at the differences between of the cubes.
For every m, $(m+1)^3-m^3 > m^3-(m-1)^3$
$9^3-8^3=217>200$

a) If m>=9 and n<=8, then $m^3-n^3>=9^3-8^3>200$. The equation can not be true.
b) If m>=9 and n>=9
   $m^3-n^3=200$, so m>n
   $m^3-n^3>=m^3-(m-1)^3>=9^3-8^3>200$. The equation can not be true.

Therefore, m,n<=8.

Try all the possible paired differences.
Let m=1, then $m^3=1$. No n can satisfy the equation.
Let m=2, then $m^3=8$. No n can satisfy the equation.
Let m=3, then $m^3=27$. No n can satisfy the equation.
Let m=4, then $m^3=64$. No n can satisfy the equation.
Let m=5, then $m^3=125$. No n can satisfy the equation.
Let m=6, then $m^3=216$. No n can satisfy the equation.
Let m=7, then $m^3=343$. No n can satisfy the equation.
Let m=8, then $m^3=512$. No n can satisfy the equation.