1. Proof: Assume $2a+3$ is rational

(Contradiction) Then $2a+3 = \frac{m}{n}$, for some integers m and n.

$$a = \frac{m-3n}{2n}$$

$m-3n$, $2n$ are integers.

Therefore a is rational, which is contradict to "a is irrational".

Q.E.D.

2. Proof: (1) $9n^3+1$ is an even integer $\longrightarrow$ n is an odd integer

(contrapositive) Assume n is an even integer.

Then $n = 2k$ for some integer k.

$9n^3+1 = 9(2k)^3+1 = 72k^3+1$.

$72k^3$ is an integer

Therefore $9n^3+1$ is an odd integer.

We've proved the negation of "$9n^3+1$ is an even integer" is true, from the assumption.

(2) n is an odd integer $\rightarrow$ $9n^3+1$ is an even integer

(Direct proof) n is an odd integer, then $n = 2p+1$ for some integer p.

$$\begin{aligned} 9n^3+1 &= 9(2p+1)^3+1 \\ &= 9\cdot(8p^3+12p^2+6p+1)+1 \\ &= 9\cdot(8p^3+12p^2+6p)+9+1 \\ &= 2\cdot\left(9\cdot(4p^3+6p^2+3p)+5\right) \end{aligned}$$

Therefore $9n^3+1$ is an even integer.

Q.E.D.