Assignment 1

1.(1) a) $(p \vee q) \wedge \neg r$

The following answer is correct as well.

$(p \oplus q) \wedge \neg r$

b) $(\neg p \wedge \neg q) \wedge r$

(2) a) $\neg((p \vee q) \wedge \neg r)$

$\equiv \neg(p \vee q) \vee \neg(\neg r)$

$\equiv (\neg p \wedge \neg q) \vee r$

Or: $\neg((p \oplus q) \wedge \neg r)$

$\equiv \neg(p \oplus q) \vee r$ ✓

$\equiv (p \leftrightarrow q) \vee r$ ✓ Note: If we use the definitions / truth table, it is easy to verify $\neg(p \oplus q) \equiv p \leftrightarrow q$

b) $\neg((\neg p \wedge \neg q) \wedge r)$

$\equiv \neg(\neg p \wedge \neg q) \vee \neg r$

$\equiv p \vee q \vee \neg r$

2. (1) Every student in our class either uses the Windows operating System or owns an iphone

(2) There is one student in our class who doesn't use the Windows operating system but owns an iphone.

3. (1) True

Assume an arbitrary real number x.

Choose $y = x - \sqrt{3}$ or $y = x + \sqrt{3}$

Then $(x-y)^2 = 3$

$\forall x \, \exists y \, ((x-y)^2 = 3)$ is True.

(2) False

If $y=2$, $z=2$, then $x=1$ can make $y/x = z$ true

If $y=2$, $z=1$, then $x=2$ can make $y/x = z$ true

Therefore, there doesn't exist x, such that $\forall y \, \forall z \, (y/x = z)$

$\exists x \, \forall y \, \forall z \, (y/x = z)$ is False

Note: You can use other examples in (2).

4. (1)

p	q	r	$q \to r$	$p \to (q \to r)$	$p \wedge q$	$\neg (p \wedge q)$	$\neg (p \wedge q) \vee r$
T	T	T	T	T	T	F	T
T	T	F	F	F	T	F	F
T	F	T	T	T	F	T	T
T	F	F	T	T	F	T	T
F	T	T	T	T	F	T	T
F	T	F	F	T	F	T	T
F	F	T	T	T	F	T	T
F	F	F	T	T	F	T	T

$p \to (q \to r)$ and $\neg (p \wedge q) \vee r$ have exact the same truth values.

(2) $p \to (q \to r)$

$\equiv \neg p \vee (q \to r)$

$\equiv \neg p \vee (\neg q \vee r)$

$\equiv \neg p \vee \neg q \vee r$

$\neg (p \wedge q) \vee r$

$\equiv \neg p \vee \neg q \vee r$

Therefore $p \to (q \to r) \equiv \neg (p \wedge q) \vee r$