MATH/CSE 1019 Test 2

Fall 2012

Nov 5, 2012

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1. (4 points) Sets. Given a set A={{∅}, {x}, 3}

   a) (1 point) Is ∅ a member of A?

      No

   b) (1 point) Is {x} a subset of A?

      No

   c) (1 points) What is the powerset of A?

      {∅, {{∅}},{{x}},{3},{{∅}, {x}},{{x}, 3},{{∅},3},{{∅}, {x}, 3}}

   d) (1 points) What is the Cartesian product of A and {b}, i.e. A × {b}?

      {<{∅},b>,<{x},b>,<3,b>}

2. (4 points) Sequences. For each of the following lists of integers, provide a simple formula that generates the terms of an integer sequence that begins with the given list.

   a) (2 points) 3, -6, 12, -24, 48, -96, …

      $a_n = 3 * (-2)^{n-1}$, n=1,2,…
      <u>Note: The sequence may start from either 0 or 1.</u>

   b) (2 points) 100, 94, 88, 82, 76, 70, …

      $b_n = 100 - 6(n-1)$, n=1,2,…
      <u>Note: The sequence may start from either 0 or 1.</u>

3. (6 points) Cardinality. Prove the following sets are countably infinite, by constructing an explicit bijective function (one-to-one correspondence) between the set of positive integers and the given set. You do not need to prove the function is bijective.

a) (3 points) the odd integers

f: Z^+ → the odd integers

f(n) = n, when n is odd
= 1-n, when n is even

b) (3 points) the integers greater than a given integer k

g: Z^+ → the integers greater than a given integer k

g(n) = n+k

4. (6 points) Functions. Let f: N × N →N be such that f(<x,y>)=x+y+1.

a) (3 points) Is this function surjective (onto)? Prove your answer is correct.

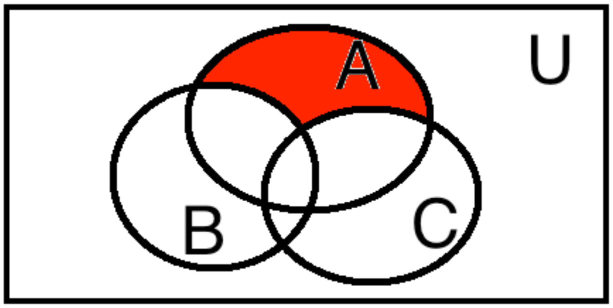
No.
Proof by giving a counter example:
f(<x,y>)=x+y+1 is always greater than or equal to 1.
Let z=0, then z∈ N, and there is no <x,y> in N × N, such that f(<x,y>)=0.
Q.E.D.

b) (3 points) Is this function injective (one-to one)? Prove your answer is correct.

No.
Proof by giving a counter example.
f(<1,2>)=1+2+1=4
f(<2,1>)=2+1+1=4
<1,2>≠<2,1>, and f(<1,2>)=f(<2,1>).
Q.E.D.

5. (5 points) Sets. Let A, B, C be sets.

a) (1 point) Draw a Venn diagram for (A-B)-C.



The red area is (A-B)-C.

b) (4 points) Prove (without using Venn diagrams) that (A-B)-C=A-(B∪C). (Hint: X-Y=X∩$\overline{Y}$.)

Note: You may prove by membership table, set identity, or ∀x(x∈((A-B)-C)≡ x∈(A-(B∪C))).

Solution 1:

A	B	C	A-B	(A-B)-C	B∪C	A-(B∪C).
0	0	0	0	0	0	0
0	0	1	0	0	1	0
0	1	0	0	0	1	0
0	1	1	0	0	1	0
1	0	0	1	1	0	1
1	0	1	1	0	1	0
1	1	0	0	0	1	0
1	1	1	0	0	1	0

(A-B)-C and A-(B∪C) have the same values.
Q.E.D.

Solution 2:

(A-B)-C

$= (A\cap\overline{B}) - C = (A\cap\overline{B})\cap\overline{C} = A\cap\overline{B}\cap\overline{C} = A\cap\overline{B\cup C}$ (De Morgan)

$= A - (B \cup C)$

Q.E.D.

Solution 3:

Let x be an arbitrary element.

x∈((A-B)-C)

≡ {x∈(A-B)∧x∉C}

≡ {x∈A∧x∉B∧x∉C}

≡ {x∈A∧x∉ $(B \cup C)$}

≡ {x∈ A-(B∪C)}

Q.E.D.

6. (5 points) Summation. Give a formula for $\sum_{i=0}^{k}\sum_{j=i}^{i+k}(3^j)$. Show your steps. You can use the following formula.

$$\sum_{i=0}^{n} ar^i = \begin{cases} \frac{ar^{n+1}-a}{r-1} & \text{if } r \neq 1 \\ (n+1)a & \text{if } r = 1 \end{cases}$$

Solution:

$\sum_{i=0}^{k}\sum_{j=i}^{i+k}(3^j)$

$=\sum_{i=0}^{k}(\sum_{j=0}^{i+k}(3^j) - \sum_{j=0}^{i-1}(3^j))$

$=\sum_{i=0}^{k}(\frac{3^{i+k+1}-1}{3-1} - \frac{3^i-1}{3-1})$

$=\sum_{i=0}^{k}(\frac{3^{i+k+1}-3^i}{2})$

$=\sum_{i=0}^{k}(3^i \times \frac{3^{k+1}-1}{2})$

$=\frac{3^{k+1}-1}{2} \times \sum_{i=0}^{k} 3^i$

$=\frac{3^{k+1}-1}{2} \times \frac{3^{k+1}-1}{3-1}$

$=\frac{1}{4}(3^{k+1}-1)^2$