## CSE4421: 5.8.4 Solution Sketch

Most students derived the forward kinematics for the bicycle. This was useful for two reasons: (1) you need the forward kinematics for question 5.8.5, and (2) you can use the forward kinematics in a simulation to cross check your solution for the inverse kinematics (and vice versa), which is what was actually required for question 5.8.4. It seems that no one tried to cross check their solutions for 5.8.4 and 5.8.5.

For question 5.8.4, you needed to derive a similar algorithm to that shown in Table 5.1; this means that you need to solve the inverse kinematics problem (i.e., given $x_{t-1}$ and $x_{t}$ find the control inputs $\hat{v}$ and $\hat{\alpha}$ needed to move from $x_{t-1}$ to $x_{t}$. You can basically use the approach described in the textbook with two important differences.

1. If you derive $\mu$ using the front wheel location $(x, y)$ then $\mu$ has terms $\cos (\theta+\hat{\alpha})$ and $\sin (\theta+\hat{\alpha})$; this does not work because you are trying to solve for $\hat{\alpha}$. You need to find another way to derive $\mu$ that does not depend on $\hat{\alpha}$.
2. The algorithm in Table 5.1 solves for the angular velocity $\hat{\omega}$ around the ICC; unfortunately, $\hat{\omega}$ is not the control input of interest in this question. You need to find a relationship between $\hat{\alpha}$ and one of the quantities calculated in the algorithm.

For question 5.8.5, you need to derive the forward kinematics for the bicycle. You can use the approach described in the slides for Day 13. First, you find the distance $R$ between the center of the front wheel and the ICC (this depends only on $L$ and $\alpha$ ). Next, you find the coordinates of the ICC (this depends on $x, y, R$, $\alpha$, and $\theta$ ). Next, you find $\omega$; it is ok if you assumed that the forward velocity of the bicycle was the velocity of the front wheel. Finally, you can find the new location and orientation of the bicycle given $x, y$, and the ICC.

