

# Lists

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# Overview

- Definition and representation of Lists in Prolog
  - Dot functor
- Examples of recursive definition of predicates
  - islist,
  - member, delete
  - append, multiple,
  - prefix, suffix, sublist

[ref.: Clocksin- Chap.3 and Nilsson- Chap. 7]

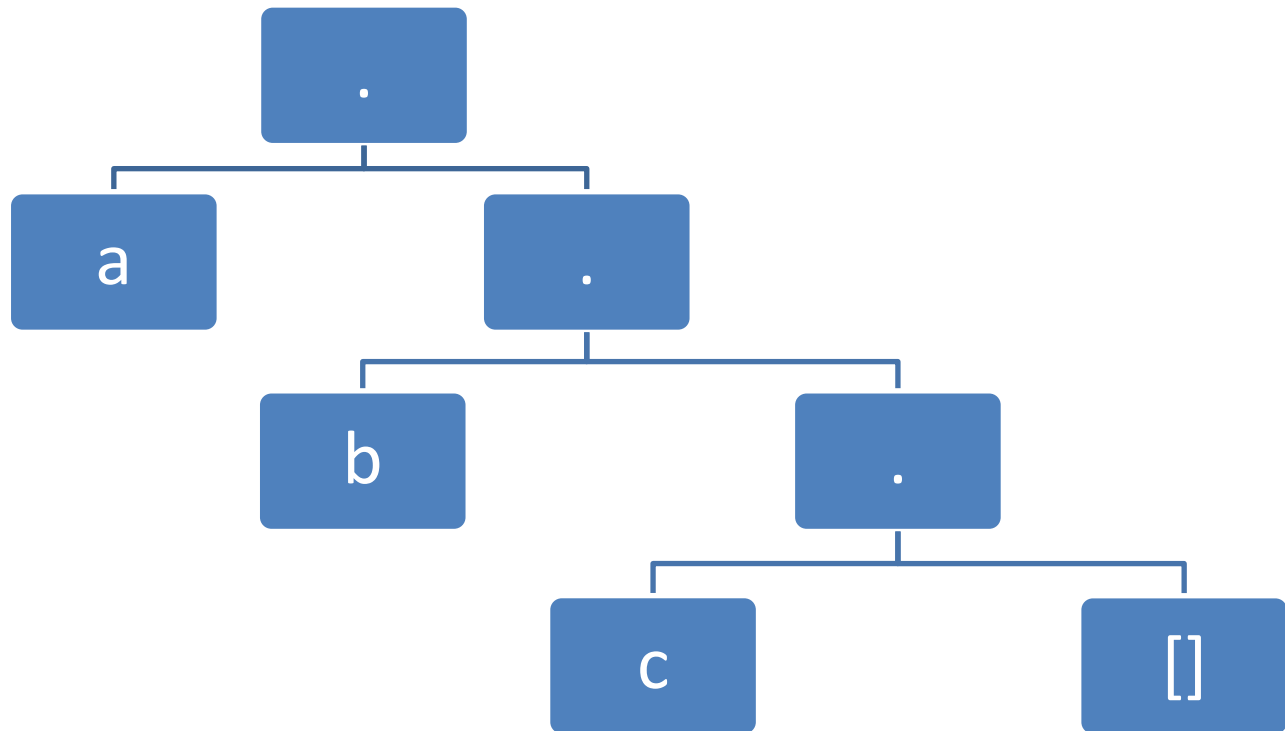
[also Prof. Gunnar Gotshalks' slides]

# Lists

- A list:
  - is an ordered sequence of elements that can have any length.
  - List notation in Prolog: [a, b, c, d, ...]
  - Either an empty list [] or it has a head X and a tail L represented as [X|L] where X is a list item and L is a list.
- The dot:
  - is a functor for representing lists with two arguments, the head and the tail of a list
  - A list of one element [a] is [a | [] ] implemented in Prolog as .(a, [])
  - [a, b] is .(a, .(b, []))
- Note [a, b, c] is not the same as [a, [b,c]]

# Lists (cont.)

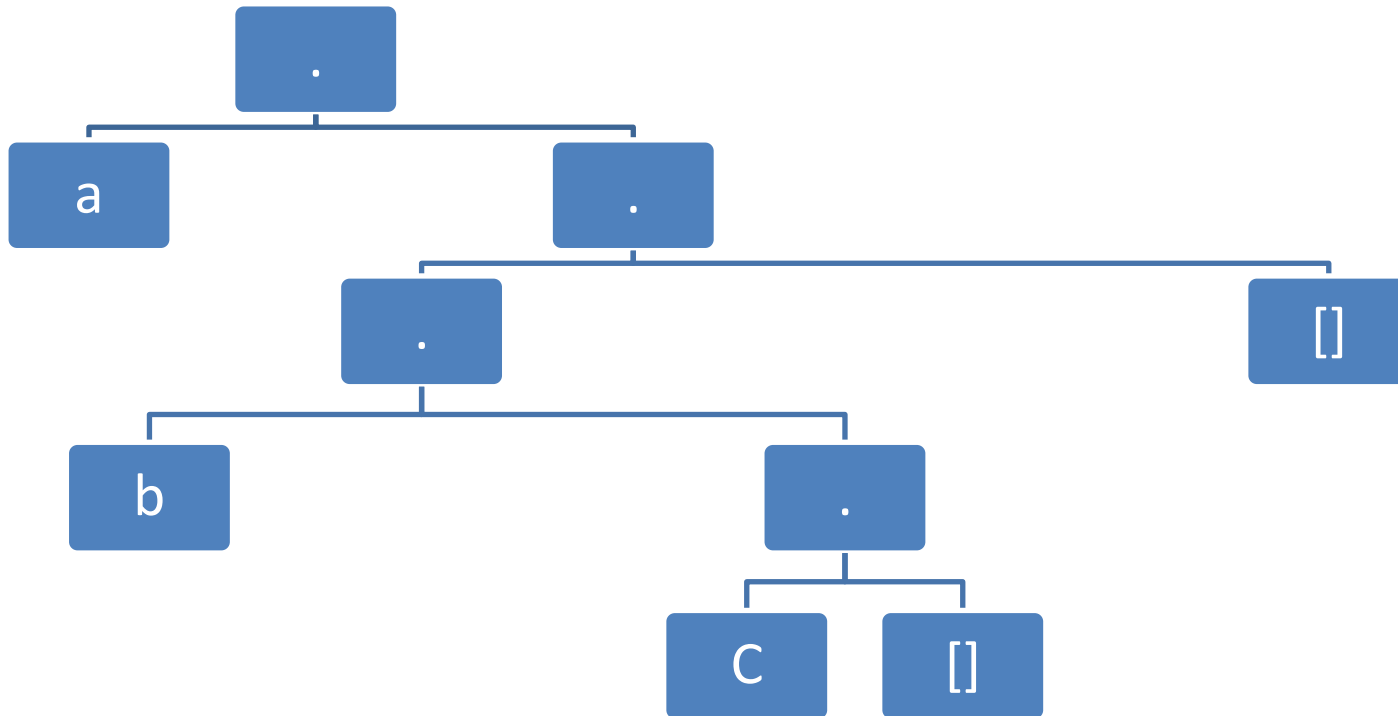
[a, b, c] is  $.(a, .(b, .(c, [])))$



# Lists (cont.)

$[a, [b, c]]$  is  $.(a, L)$

where  $L$  is  $[[b,c]]$  having  $[b,c]$  as its head and  $[]$  as tail



# Examples

- Write the Prolog definition for being a list.

```
islist([]).
```

```
islist([Head | Tail]) :- islist(Tail).
```

- Write the Prolog definition for being a member of a list.

```
member(X, [X | L]).
```

```
member(X, [Y | L]) :- member(X,L).
```

# Examples (cont.)

```
:- member(3, [2, 3, 4, 5]).  
true
```

```
:- member(3, [2, [3, 4], 5]).  
false
```

Our definition does not consider members of members (nested lists)

```
:- member(X, [1, 2]).  
X = 1 ;  
X = 2 ;  
false
```

Unlike other programming languages, inputs can be unknowns

```
:- member(2, L).  
L = [2 | _] ;  
L = [_ , 2 | _] ;  
...
```

Note the recursive definition of member

# Recursive Definition

- Example:
  - member(X, [X | L]). : boundary condition
  - member(X, [Y | L]) :- member(X,L). : recursive case
- Note in a recursive definition, the problem must get smaller in each recursion to guarantee convergence
  - member(X,L) is a smaller problem than member(X, [Y | L])
- Prolog does not need to remember Y, so we can use the **anonymous variable** `_` instead:
  - member(X, [\_ | L]) :- member(X,L).



# Recursive Search

- Example:  
member(X, [X|L]).  
member(X, [Y|L]) :- member(X,L).

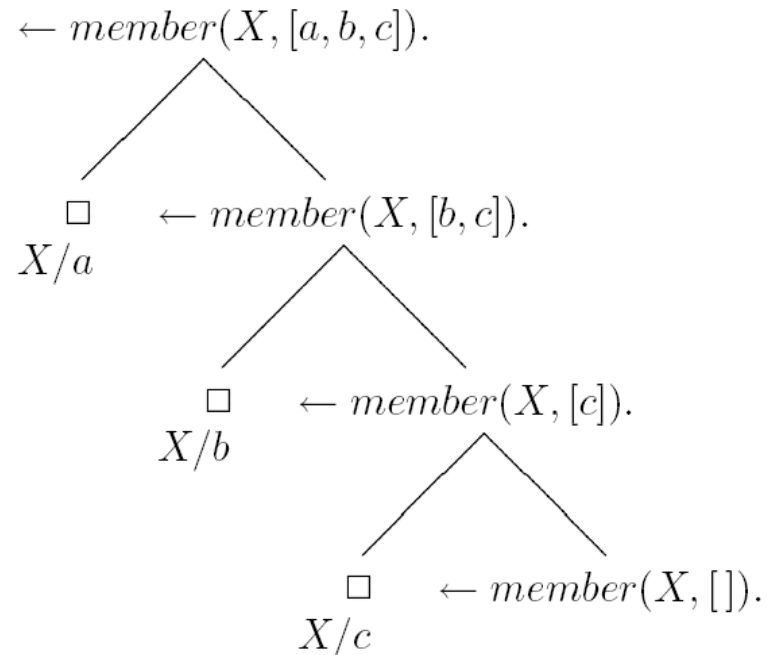
`:-member(X, [a,b,c]).`

`X = a;`

`X = b;`

`X = c;`

`false`



# Delete

- `delete(X, L1, L2)` is true if `L2` is the result of deleting `X` from `L1` (just once).
  - For example: `delete(5, [1, 5, 4, 2], [1, 4, 2])`.

*`delete(X, [X|L], L)`.*

*`delete(X, [Y|L], [Y|L1]) :- delete(X, L, L1)`.*

# Append

- Join two lists:

Example: `append([1,2], [3,4], [1,2,3,4])`

`append([], L, L).`

`append([X|L1], L2, [X|L3]) :- append(L1, L2, L3).`



: boundary condition

: recursive case

a smaller problem

- Possible Queries:

[Nilsson]

`:- append([a, b], [c, d], [a, b, c, d]).`

true

`:- append([a, b], [c, d], X).`

`X=[a, b, c, d]`

or even

`:- append(Y, Z, [a, b, c, d]).`

# Search tree for append query

append([], X, X).

append([X|Y], Z, [X|W]) :-

append(Y, Z, W).

:- append(Y, Z, [a, b, c, d]).

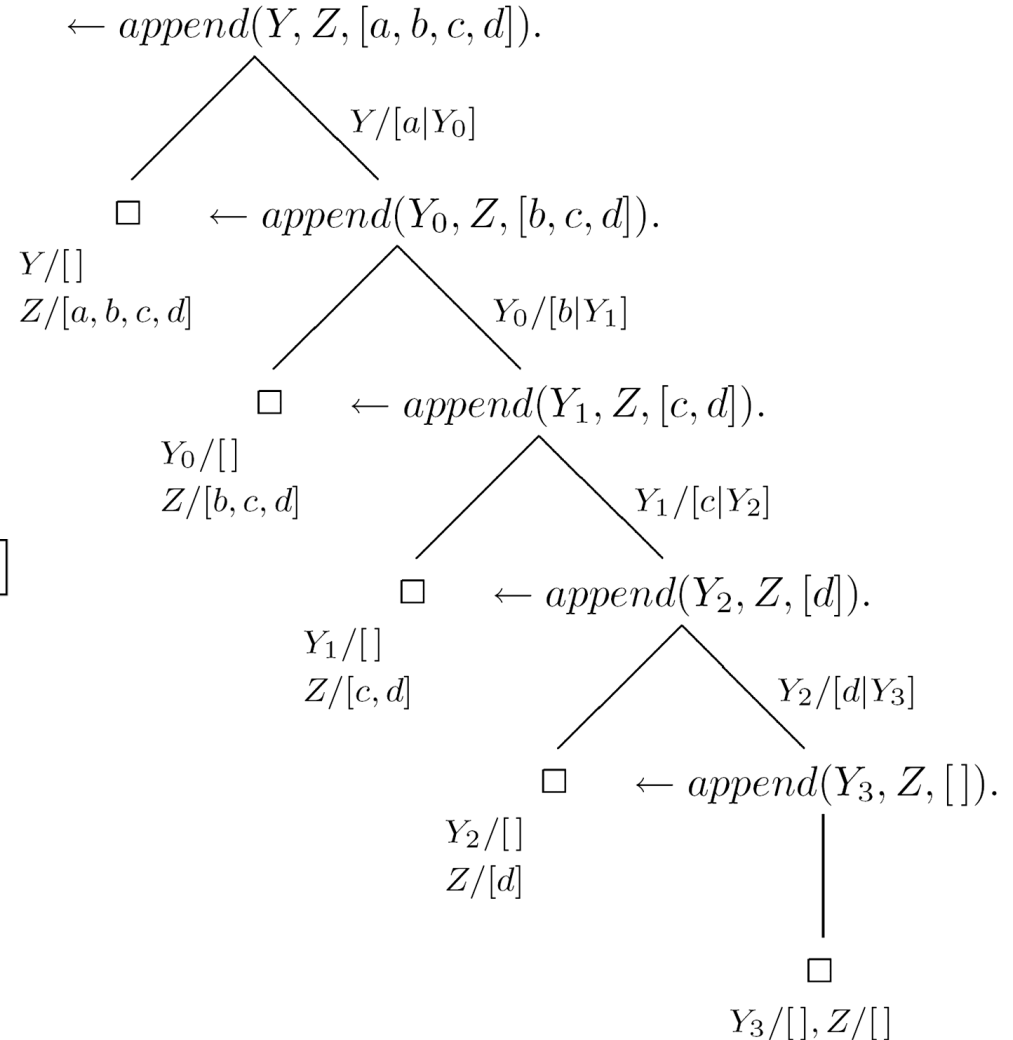
$Y = []$                        $Z = [a, b, c, d]$

$Y = [a]$                        $Z = [b, c, d]$

$Y = [a, b]$                        $Z = [c, d]$

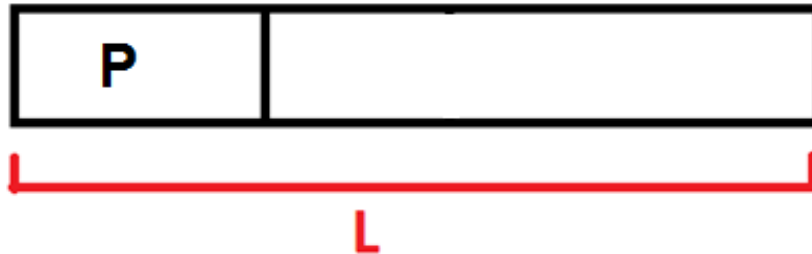
$Y = [a, b, c]$                        $Z = [d]$

$Y = [a, b, c, d]$                        $Z = []$



# Prefix with append

- The list  $P$  is a prefix of  $L$ , if  $L$  can be obtained by appending  $P$  to another list.



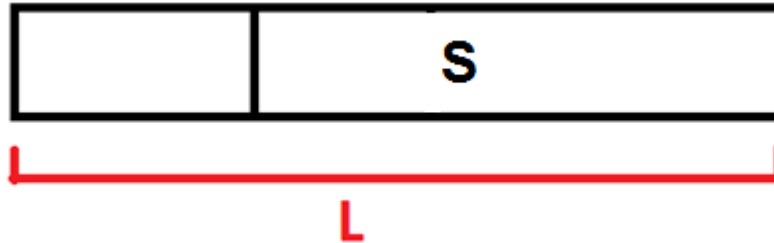
- Write  $\text{prefix}(P, L)$  which is true if  $P$  is a prefix of  $L$ .

*prefix(P, L):- append(P, \_, L).*

– Is  $[]$  a prefix of  $L$ ?

# Suffix with append

- The List S is a suffix (or postfix) of L if L can be obtained by appending some other list with S.



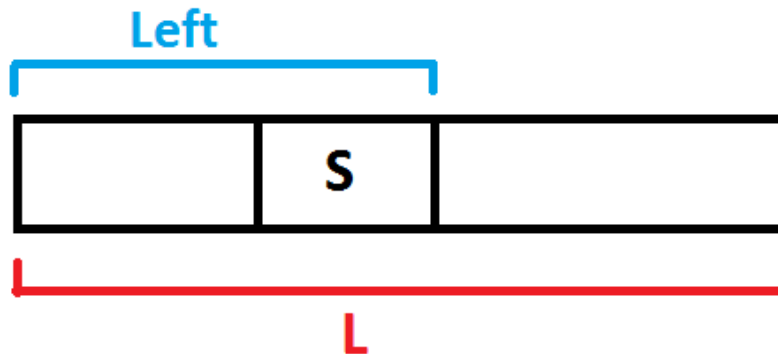
- Write `suffix(S,L)` which is true if S is a suffix of L

*suffix(S,L):- append(\_, S, L).*

- Exercise: Try writing prefix and suffix without using append.

# More Examples with append

- `sublist(S,L)` is true if `S` is a sublist of `L`
  - in other words, `S` is the suffix of a prefix

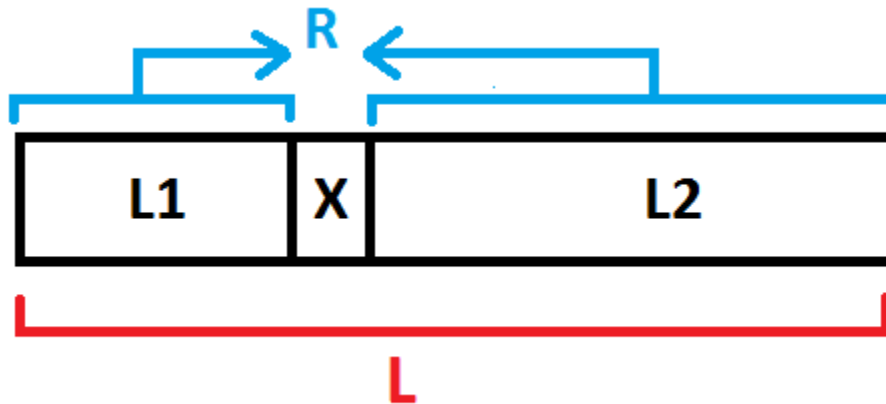


– Using `append(..)`:

*`sublist(S,L):- append(_, S, Left), append(Left, _, L).`*

# More Examples with append

- Re-writing `delete(X,L1,L2)` with `append(..)`:



*`delete(X, L, R):- append(L1,[X|L2],L), append(L1, L2, R).`*



# Example: multiple occurrences in a list

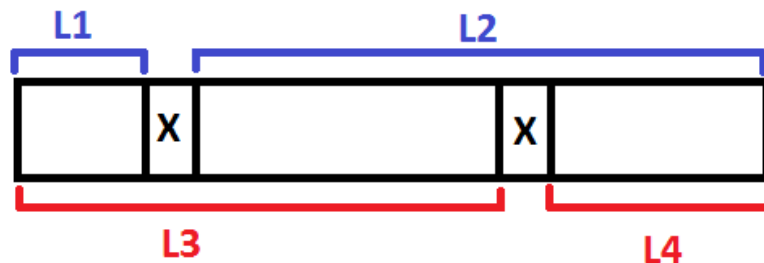
- `multiple(L)` is true if `L` is a list with multiple occurrences of some element [Nilsson]:

*`multiple([Head|Tail]):- member(Head, Tail).`*

*`multiple([Head|Tail]):- multiple(Tail).`*

– Writing `multiple(..)` using `append(..)`

*`multiple(L) :- append(L1, [X|L2], L), append(L3, [X|L4], L).`*



What is missing in definition of `multiple(..)`? How can it be corrected?

# Append is expensive!

*append([], L, L).*

*append([X/L1], L2, [X/L3]) :- append(L1, L2, L3).*

- The complexity of appending two lists, L1 and L2, is  $O(n)$  where  $n$  is the length of the first list.
- Consider *reverse(L, R)* defined as:  
*reverse([], []).*  
*reverse([X/L], R) :- reverse(L, L1), append(L1, [X], R).*
- Complexity of *reverse(..)* is  $O(n^2)$  where  $n$  is the length of  $L$ .