Homework Assignment #9
Due: March 26, 11:30 a.m.

This problem set has two questions that can be solved using dynamic programming. If you provide a solution using dynamic programming, then your solution should include the following items.

- A clear definition of the array you are using (including its dimensions and a clear English description of what the value stored in each element of the array is supposed to represent).
- The equations you use to fill in the array, together with some explanation of why the equations are correct.
- Pseudocode that fills in the array.
- Pseudocode that uses the array to print the required output.

1. A deterministic finite automaton $F = (Q, \Sigma, q_0, \delta, F)$, is specified by the set of states $Q = \{q_0, q_1, q_2, \ldots, q_m\}$, the input alphabet $\Sigma = \{a_1, a_2, \ldots, a_\ell\}$, the initial state $q_0 \in Q$, the transition function $\delta : Q \times \Sigma \to Q$ and the set of accepting states $F \subseteq Q$. Given F and a non-negative integer n, determine the number of strings of length n that the automaton accepts.

If the automaton F is fixed, your algorithm should run in $O(n)$ time (assuming all numbers fit into a single word). You will get bonus marks for a correct algorithm that is even faster (as a function of n, when F is fixed).

2. You are driving a truck from Toronto to Vancouver, a distance of 4500 kilometres. You start in Toronto with a full tank of gas. Your truck's gas tank can hold 200 litres of gas. The truck can travel 1 kilometre on each litre of gas (it has lousy fuel efficiency). There is no constraint on how much gas should be in your tank when you arrive in Vancouver. You have chosen the route you will travel, and you have a map of the n gas stations along the route. You have also found a web site that lists the gas prices at every gas station.

Let d_i be the distance from your starting point to the ith gas station along the route in kilometres and let p_i be the price of gas per litre (in 10ths of a cent) at the ith gas station. (The gas stations are listed in order so that $d_1 < d_2 < \cdots < d_n$ and for all i, $d_{i+1} - d_i \leq 200$.) Each d_i and p_i is an integer. Each time you make a gas purchase (using your debit card), your bank charges you a \$1.50 transaction fee. You may assume that each purchase is an integral number of litres. (Because of the way the problem is set up, there is no advantage to purchasing a fraction of a litre.)

Your goal is to get from Toronto to Vancouver spending as little money on gas as possible. Design an efficient algorithm to print a list describing how many litres of gasoline you should buy at each gas station along the route.