Homework Assignment #5
Due: February 15, 11:30 a.m.

1.

(a) Suppose $T(n,k)$ satisfies the following relationships.
$T(n,0) = 1$
$T(n,1) = n$
$T(n,k) \leq \max\limits_{0 \leq x \leq n} \left(T(x, \left\lfloor \frac{k}{2} \right\rfloor) + T(n-x, \left\lceil \frac{k}{2} \right\rceil) + cn \right)$, for $k \geq 2$,
where c is a constant. Prove that there is a constant d such that for all $k \geq 2$ and for all n, $T(n,k) \leq dn \log_2 k$.

(b) The element of rank i in an array $A[1..n]$ is the ith smallest element of A.

Design an algorithm Choose$(A[1..n], b, k)$ where b is a positive integer and k is a non-negative integer such that $kb \leq n$. The algorithm should print the elements of A whose ranks in A are $b, 2b, 3b, \ldots, kb$ (in order). (Note that when $k = 0$, your algorithm should print nothing.) Your algorithm should run in $O(n \log k)$ time. For simplicity, you can assume that all the elements in A are distinct.

You do not need to provide a full, formal proof of correctness, but you should explain why your algorithm is correct and why it runs in $O(n \log k)$ time. You should also include precise pre- and post-conditions for any routine that you write, and loop invariants for any loops.

Hint: design a divide-and-conquer algorithm and use part (a).