Homework Assignment #3
Due: January 30, 11:30 p.m.

1. Let $B(x)$ denote the binary representation of the positive integer x (with no leading 0's). Formally, this means that $B(x)$ is a string $b_\ell \dots b_0$ where
(1) $\sum_{i=0}^{\ell} b_i 2^i = x$,
(2) $b_i \in \{0, 1\}$ for $0 \leq i < \ell$, and
(3) $b_\ell = 1$.

In class, we saw an algorithm that used a loop to output the binary representation of a natural number. Here is a recursive algorithm that accomplishes the same task.

BINARY(x)
  Precondition: x is a positive integer
  Postcondition: prints string $b_\ell \dots b_0$ satisfying (1)-(3) above.
  if $x \geq 2$ then
    BINARY($\lfloor \frac{x}{2} \rfloor$)
  end if
  print x mod 2
end BINARY

(a) Give a careful proof that this algorithm is correct.

(b) Use big-Theta notation to describe the worst-case running time of this algorithm. Specify what quantity you are using to measure the size of the input. Show your answer is correct.