Homework Assignment #2
Due: January 23, 11:30 p.m.

1. $\binom{n}{k}$, pronounced "n choose k", denotes the number of subsets of $\{1, 2, \ldots, n\}$ that contain exactly k elements, where n and k are natural numbers. (I use the convention that the natural numbers include 0.) The value of $\binom{n}{k}$ can be defined recursively using Pascal's Rule.

$$\begin{aligned}\binom{n}{0} &= 1, \text{ for } n \geq 0\\ \binom{0}{k} &= 0, \text{ for } k > 0\\ \binom{n}{k} &= \binom{n-1}{k-1} + \binom{n-1}{k}, \text{ for } n > 0 \text{ and } k > 0\end{aligned}$$

We can compute $\binom{n}{k}$ by filling in a 2-dimensional array $C[0..n, 0..k]$ such that $C[i,j] = \binom{i}{j}$. We fill in the array row-by-row, going left to right in each row. To fill in each entry, we use the recursive definition above. This gives us the following algorithm.

```
CHOOSE(n, k)
    Precondition: k, n ∈ ℕ and 0 ≤ k ≤ n
    Postcondition: The output is (n choose k)
    i = 0 // row index
    j = 0 // column index
    loop
        exit when i > n
        // compute C[i, j] = (i choose j)
        if j == 0 then C[i, j] = 1
        elseif i == 0 then C[i, j] = 0
        else C[i, j] = C[i − 1, j − 1] + C[i − 1, j]
        end if
        j = j + 1 // advance to next position in row
        if j > k then // arrived at end of row
            i = i + 1 // start filling in next row
            j = 0
        end if
    end loop
    return C[n, k]
end CHOOSE
```

Give a formal proof of termination. (You do *not* have to prove the postcondition.)

Hint: find some measure of progress that is an integer and increases in every loop iteration, and show that when that measure becomes large enough, the loop terminates.