

**Test 2****First Name:** \_\_\_\_\_**Last Name:** \_\_\_\_\_**Student Number:** \_\_\_\_\_

*This test lasts 75 minutes. No aids allowed.*

*Make sure your test has 5 pages, including this cover page.*

*Answer in the space provided. (If you need more space, use the reverse side of the page and indicate **clearly** which part of your work should be marked.)*

*Please write legibly.*

Question 1	/1
Question 2	/3
Question 3	/2
Question 4	/3
Question 5	/3
Question 6	/3
Question 7	/3
Question 8	/3
Total	/21

- [1] 1. Who invented Turing machines?
- [3] 2. Give a regular expression for the set of all binary strings that contain at least two 0's. (The two 0's need not be consecutive.)
- [2] 3. State the Church-Turing Thesis in your own words.
- [3] 4. Given regular expressions  $R_1$  and  $R_2$  for languages  $L_1$  and  $L_2$ , describe an algorithm that would construct a regular expression for  $L_1 \cap L_2$ .  
You may use any algorithm given in the textbook or in the lectures as a subroutine (without having to describe how that subroutine works).

- [3] 5. Is it true that every subset of every decidable language is also decidable? Circle the correct answer and then prove your answer is correct.

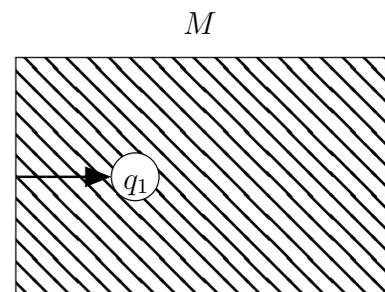
YES

NO

- [3] 6. Let  $M$  be a Turing machine with input alphabet  $\Sigma = \{0, 1\}$  and initial state  $q_1$ . Assume that its transition diagram is inside the shaded box below. (The details of its states and transitions are not shown in the picture, and you do not need to know what they are).

You wish to design another Turing machine  $M'$ , which also has input alphabet  $\{0, 1\}$ . When  $M'$  is given an input string  $z$ , it is supposed to erase  $z$ , write 01 on the tape and then simulate the actions of  $M$  on the input string 01.

Draw some additional states and transitions (outside the box) to accomplish this.



- [3] 7. We call a Turing machine  $M$  *slow* if there exists a string  $w$  such that  $M$  takes at least  $|w|^2$  steps when it is given input string  $w$ . Describe an algorithm that *recognizes* the language  $SLOW_{TM} = \{\langle M \rangle : M \text{ is a slow Turing machine}\}$ .

- [3] 8. Let  $L = \{\langle M \rangle : M \text{ is a Turing machine that does not accept } \varepsilon\}$ .  
Prove that  $L$  is undecidable.