YORK UNIVERSITY

FACULTY OF SCIENCE AND ENGINEERING

2011 WINTER TERM EXAMINATION

Course Number: CSE2001

Title: Introduction to Theory of Computation

Duration: 3 hours

No aids allowed.

- There should be 10 pages in the exam, including this page.
- Write all answers on the examination paper. If your answer does not fit in the space provided, you can continue your answer on the back of a page or on page 10, indicating clearly that you have done so.
- You may use Church's Thesis in your answer to any question.
- Write legibly.

Name ____

(Please underline your family name.)

Student Number __________________________________

1.	____________/8
2.	____________/3
3.	____________/3
4.	____________/4
5.	____________/3
6.	____________/2
7.	____________/3
8.	____________/3
9.	____________/4
10.	____________/4
11.	____________/4
12.	____________/5
13.	____________/4
Total:	____________/50

1. [8 marks] For each of the following languages, you must determine whether the language is regular, context-free, decidable, recognizable or not recognizable. For each language, circle the *leftmost* correct answer. For example, if a language is both recognizable and decidable, but not context-free, circle decidable.

Recall that w^R denotes the reverse of string w. For example, if $w = 1011$, then $w^R = 1101$.

In the following, $B(n)$ denotes the binary representation of the integer n (with no leading 0's). For example, $B(23) = 10111$.

(a) $\{w \in \{0,1\}^* : w = w^R\}$.

regular   context-free   decidable   recognizable   not recognizable

(b) $\{B(n)\#B(n+1) : n \in \mathbb{N}\}$. The alphabet for this language is $\Sigma = \{0,1,\#\}$.

regular   context-free   decidable   recognizable   not recognizable

(c) $\{(B(n))^R\#B(n+1) : n \in \mathbb{N}\}$. The alphabet for this language is $\Sigma = \{0,1,\#\}$.

regular   context-free   decidable   recognizable   not recognizable

(d) $\{w \in \{0,1\}^* : w$ contains an even number of 0's and an odd number of 1's$\}$.

regular   context-free   decidable   recognizable   not recognizable

(e) $\{0^i 1^j 2^i 3^j : i \geq 0$ and $j \geq 0\}$.

regular   context-free   decidable   recognizable   not recognizable

(f) $\{w \in \{0,1\}^* : w$ contains more 0's than 1's$\}$.

regular   context-free   decidable   recognizable   not recognizable

(g) $\{\langle M \rangle : M$ is a Turing machine and there is a string w such that M takes at least $|w|+3$ steps on input $w\}$.

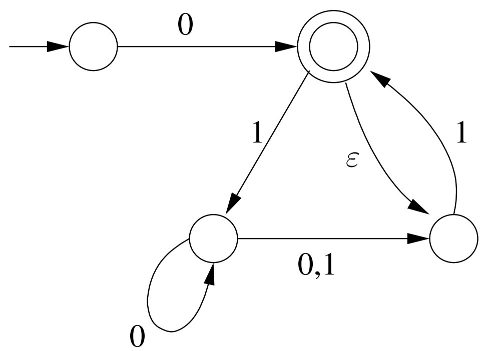
regular   context-free   decidable   recognizable   not recognizable

(h) $\{\langle M \rangle : M$ is a Turing machine and for every input string w, M takes at least $|w|+3$ steps on input $w\}$.

regular   context-free   decidable   recognizable   not recognizable

2. [3 marks] Let $L_2 = \{w \in \{0,1\}^* :$ there is a 1 in every odd position of $w\}$. Draw the transition diagram of a deterministic finite automaton for L_2.

3. [3 marks] Give a regular expression for the language accepted by the nondeterministic finite automaton shown below.



4. [4 marks] Let $L_4 = \{0^i 1^j : i \geq 0, j \geq 0 \text{ and } i > 3j\}$. Give a context-free grammar for L_4.
(You do *not* have to prove that your answer is correct.)

5. [3 marks] Recall that if w is a string, then w^R denotes the reverse of w. Suppose $G = (V, \Sigma, R, S)$ is a context-free grammar for a language L. Describe how to construct a grammar G' for the language $L^R = \{w^R : w \in L\}$.
(You do *not* have to prove that your answer is correct.)

6. [2 marks] Explain the difference between a Turing machine that recognizes a language L and a Turing machine that decides the language L.

7. [3 marks] Give a precise definition of $L \leq_m L'$. (I.e., what does it mean for a language L to be many-one reducible to a language L'?)

8. [3 marks] Is it true that every subset of every undecidable language is also undecidable? Circle the correct answer and then prove your answer is correct.

YES NO

9. [4 marks] In this question, the alphabet we are considering is $\Sigma = \{0, 1, \#\}$.
Let $L_9 = \{x\#y : y \in \{0,1\}^* \text{ and } x \text{ is a substring of } y\}$. For example, $1011\#1010110 \in L_9$ since 1011 is a substring of 1010110. Prove that L_9 is not regular.

10. [4 marks]
Let $L_{10} = \{\langle M \rangle : M$ is a Turing machine that accepts some string that contains 111 as a substring$\}$.
Show that L_{10} is recognizable.

11. [4 marks]
Let $L_{11} = \{\langle R \rangle : R$ is a regular expression that generates some string that contains 111 as a substring$\}$.
Sketch an algorithm that decides L_{11}.

12. [5 marks] Let $L_{12} = \{w \in \{0,1\}^* : w \text{ has odd length and the middle character of } w \text{ is } 1\}$. Give a formal proof that the following grammar (with starting symbol S) generates every string in L_{12}.

$$
\begin{array}{lllllll}
S & \rightarrow & TST & \qquad & T & \rightarrow & 0 \\
S & \rightarrow & 1 & \qquad & T & \rightarrow & 1
\end{array}
$$

If you use a proof by induction, clearly state the claim that you are proving by induction.

13. [4 marks] Let $L_{13} = \{\langle M, w\rangle : M$ is a Turing machine, w is a string and M accepts neither w nor $w^R\}$. Show that L_{13} is undecidable.

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