Homework Assignment #6
Due: June 25, 2012 at 7:00 p.m.

1. A 2-dimensional Turing machine (2DTM) uses a 2-dimensional blackboard instead of a 1-dimensional tape. The blackboard is of unbounded size and is divided into squares, each of which can hold a single character from the machine's alphabet Γ. At each step, a 2DTM chooses what to do next based on its state and the character written at the machines current location on the blackboard. The 2DTM can change state, replace the character at its current location with a new character and move north, south, east or west one square. Initially, the input string is stored in a sequence of squares stretching to the east from the Turing machine's initial location on the blackboard.

We wish to show that 2DTMs are no more powerful than ordinary TMs by showing how simulate a 2DTM with an ordinary TM. We shall use a multitape TM to perform the simulation, since we already know how to simulate a multitape TM with a single-tape TM.

In the multitape machine, tape 1 will store the input string, tape 2 will store the contents of the simulated blackboard, tape 3 will store the x-coordinate of the simulated machine (in binary), and tape 4 will store the y-coordinate of the simulated machine (in binary).

Tape 2 uses the following data structure to store the contents of the blackboard. The tape contains a list of triples of the form (x, y, z), where x and y are numbers represented in binary and z is a single character in Γ. Each triple (x, y, z) indicates that the square on the blackboard with x-coordinate x and y-coordinate y contains character z. The triples are not stored in any particular order. Assume the coordinates of the square where the 2DTM begins are $(0, 0)$.

(a) Suppose a tape stores a positive number in binary representation. There is nothing else on the tape, except blank squares. Give instructions that a Turing machine could follow to add one to the number that is stored on the tape.

(b) Describe, at a high level, how a Turing machine could add one to an integer stored on a tape in binary representation if the integer could be positive, negative or zero.

(c) Describe, at a high level, how to set up the contents of the 4 tapes before beginning to simulate steps of the 2DTM. (Assume, as usual, that the input to the multitape TM is initially written on the first tape and all other tapes are blank.)

(d) Describe, at a high level, how to simulate one step of the 2DTM using the 4-tape TM outlined above.

For part (a), give a fairly detailed description of the steps the Turing machine should follow (similar to the steps used to describe M_4 on page 147 of the textbook). For parts (b), (c) and (d) the level of detail in your answer should be similar to the description of D at the bottom of page 151.