Homework Assignment #1
Due: May 14, 2012 at 7:00 p.m.

This assignment is based on material that is a prerequisite to this course. See the handout "Mathematical Prerequisites". After doing any rough work required, you should write your solutions on this document in the spaces provided and then hand it in. (Most of the assignments later in this course will have fewer, but longer, questions; this assignment is just intended to give you quick feedback on whether you need to review some of the background material more thoroughly.)

Name: ____

Student Number: ___________________________

1. The last two digits of your student number form a decimal number between 0 and 99. Write down the binary representation of that number.

2. Let the predicate $E(x, y)$ represent the statement "Person x eats food y".

(a) Express the following statement in predicate logic: "There is some food that John eats, but nobody else does."

(b) Express the following statement in predicate logic: "John eats nothing but hot dogs."

(c) Explain the difference between the statements "$\forall x \exists y E(x, y)$" and "$\exists y \forall x E(x, y)$".

3. For each of the following sets, determine whether the set is finite or infinite. If the set is finite, write down an explicit list of all the elements in the set. If the set is infinite, say so and list five elements of the set. (Assume $0 \in \mathbb{N}$.)

(a) $A = \{n \in \mathbb{N} : \exists m \in \mathbb{N} \text{ such that } 2n = 3m\}$.

(b) $B = \{2n : n \in \mathbb{N} \text{ and } n < 4\}$.

(c) $C = \{(n, m) \in \mathbb{N} \times \mathbb{N} : 2n = m\}$.

(d) $D = \{-1, 2\} \times \{0, 1\}$.

(e) $E = \mathscr{P}(\{0, 2, 9\})$

(f) $F = \{\} \times \mathbb{N}$

4. If $w : \{1..\ell\} \to \Sigma$ is a string, the reverse of w, denoted w^R, is the string of length ℓ defined by $w^R(i) = w(\ell+1-i)$. If L is a language, the reverse of L is the language $L^R = \{w^R : w \in L\}$. Use these definitions to give careful proofs of the following statements.

(a) For every string x, if $(x^R)^R = x$.

(b) For every language L, if $L \subseteq L^R$, then $L = L^R$.