Homework Exercise #3
Due: October 6, 2011

3. Assume we have an undirected, connected network graph.

(a) Recall the basic flooding algorithm for broadcast in an asynchronous message-passing system. To initiate a broadcast(msg), a process sends the message m to each of its neighbours. Whenever another (non-initiator) process receives msg for the first time, the process immediately sends msg to every neighbour except the one from which it received the message. If any process receives msg again after it has already sent it, the process ignores the message. (You may assume if a process receives two messages at exactly the same time, the process first handles one of them according to the algorithm, and then the other. In other words, the receiver pretends that one arrived slightly before the other.)

We want to figure out the exact message complexity of the algorithm, when it is used by a single broadcaster to broadcast a single message. Give a function $f(n, m)$ such that, for every graph that has n nodes and m edges, exactly $f(n, m)$ messages are sent in every execution of the basic flooding algorithm. Do not use asymptotic (big-O) notation to define your function f; I want the exact value. Prove that your answer is correct.

(b) Now consider the variant of the flooding algorithm in which acknowledgements are sent (so that the broadcast initiator knows when every process has received the message). It works as follows.

Each process uses a parent pointer to point to one of its neighbours. Initially each pointer is null. These pointers will eventually form a tree, with all pointers pointing towards the root of the tree.

To initiate a broadcast(msg), a process sets its parent pointer to itself (to indicate that it is the root of the tree) and sends the message msg to each of its neighbours. It knows the broadcast is complete when it has received an acknowledgement message ack from all of its neighbours.

Whenever another (non-initiator) process receives msg for the first time, it immediately sets its parent pointer to the process from which it received msg and sends msg to each of its neighbours, except its parent. Later, when it has received an ack from all of its neighbours except its parent, it sends an ack to its parent.

If any process receives msg from a neighbour after its parent pointer is set, it immediately sends back an ack message.

Give a function $g(n, m)$ such that, for every graph that has n nodes and m edges, exactly $g(n, m)$ messages are sent in every execution in which this algorithm is used by a single broadcaster to broadcast a single message. Prove that your answer is correct.