

Homework Exercise #1

Due: September 22, 2011

Policy on homework assignments: The solutions you hand in should be your own work. Although you may discuss the *general* approach to solving a problem with others, you should not discuss the solution in detail. You must not take any written notes away from such a discussion. Also, you must list on the cover page of your solutions any people with whom you have discussed the problems. If you get stuck while working on one of the assignments, I encourage you to come to my office hours to get help with it instead of consulting outside sources. However, if you do use ideas from any source other than the lecture notes (e.g., a paper or a web page), you should cite your source properly. If you have questions about Academic Honesty you can consult the following web page: <http://www.cse.yorku.ca/admin/coscOnAcadHonesty.html> .

1. Recall the Two Generals problem from class. The generals of the red army have synchronized watches. Each red general has an initial preference for what time to attack. Each red general must eventually decide what time to lead his troops in an attack. The decisions must satisfy the following two properties.

Agreement property: In every possible execution of their plan, the decisions of the two red generals must be identical.

Validity property: If all generals have the same initial preference x and no communication failures (described below) occur, no general should decide on a time different from x .

The red army's messengers have pooled their money to buy two walkie-talkies, one for each of their generals. (They figure that this will make it unnecessary for them to perform the dangerous task of carrying messages between the generals.) Each walkie-talkie has a switch that allows it to be set either to send mode or to receive mode. When a walkie-talkie is in send mode, it does not output any sound. If both walkie-talkies are in receive mode, they both output white noise. If one walkie-talkie is in send mode and the other is in receive mode, sounds picked up by the microphone in the first one are output by the second one. The walkie-talkies are the only means of communication available to the red generals.

However, the enemy blue army has a jamming device that can be used to disrupt the behaviour of walkie-talkies that are in receive mode. (The device is basically a powerful transmitter that overwhelms the weak signals sent by the walkie-talkies.) When the jamming device is transmitting, it prevents communication between the walkie-talkies as described below. The jamming device is a complicated piece of machinery. When it is switched on, it takes exactly three seconds before it warms up and begins transmitting its jamming signal (and it cannot be turned off during those three seconds). When it is switched off, it also takes exactly three seconds before the jamming signal ceases (and it cannot be turned back on during those three seconds). Assume the jamming device is initially off.

In this context, the validity property says that if the two generals have the same initial preference x and the blue army never turns on its jamming device, then neither general should decide on a time different from x .

- (a) Suppose that the jamming device works by sending out a strong, steady B-flat tone (440 Hz). Thus, if a walkie-talkie is in receive mode when the jamming device is transmitting, then the walkie-talkie will only output a steady B-flat note. Is the Two Generals problem solvable? Prove your answer is correct.
- (b) Now suppose that the jamming device works by sending out a white noise signal. Thus, if a walkie-talkie is in receive mode when the jamming device is transmitting, then the walkie-talkie will only output white noise.

For either part, if you wish to show that the problem is solvable, give an algorithm that solves it and explain why it satisfies the problem specification. Be specific about the exact steps each general must perform (e.g., “At 7:00 am, switch your walkie-talkie into send mode and whistle the *Liechtensteiner Polka* for 2 seconds.”). If you wish to show the problem is unsolvable, you must give an argument that proves no possible algorithm can satisfy the specification.