CSE4421: Assignment 3

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1. For both the Bug1 and Bug2 algorithms, derive the upper bound on the distance travelled by the robot. You might find the following parameters useful in your derivation:

- D the distance between the start and goal points.

- P_i the perimeter of the ith obstacle encountered by the robot.

- for Bug2, n_i the number of times the m-line intersects the ith obstacle encountered by the robot.

You must explain your derivation (instead of simply writing down the upper bound equations). Note that the equation given on page 202 for Bug1 is incorrect.

2. Consider a bicycle with two steerable wheels as shown in the following figure. The front and back wheels of the bicycle are connected by a frame of length L making an angle θ relative to the world X-axis. The front wheel is powered having a ground speed v_F and making an angle α_F with the bicycle frame. The rear wheel makes an angle α_R with the frame. The origin of the bicycle is located on the axle of the rear wheel.
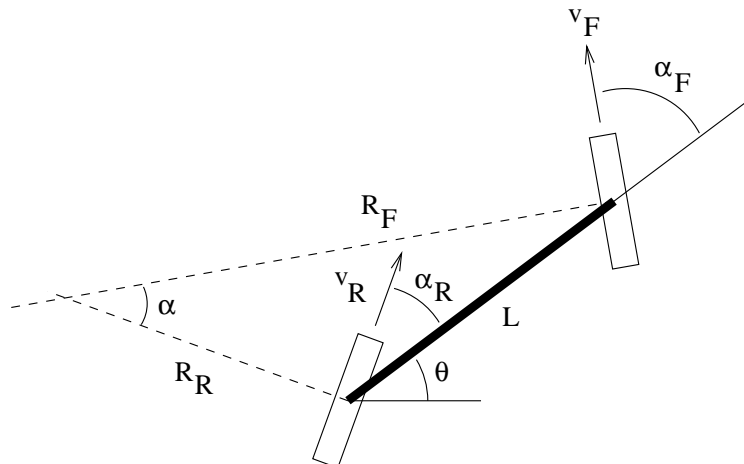
(a) Find an expression for the angle α in terms of α_F and α_R.

(b) Find expressions for R_F and R_R in terms of L, α_F, and α_R.

(c) What ground speed v_R of the rear wheel is required for smooth rolling motion?

3. Suppose you have a mobile robot with a range sensor that measures the location of a point on a solid object; its measurement noise (assumed to be additive zero-mean Gaussian) is smaller in the direction perpendicular to the object surface than in the direction tangential to the surface as shown in the figure below (the shaded ellipse is the covariance ellipse of the measurements).
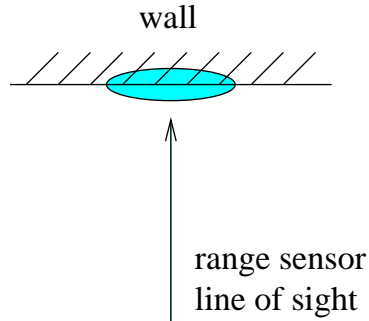
The file /cs/dept/www/course/4421/assignments/a3.mat (also available from the course web site) contains 100 sample measurements of a single point on a wall located in some world coordinate frame; the measurements are in the world frame (not the robot frame). Copy the file into a working directory and load its contents into Matlab by typing

```
load a3.mat
```

The file contains a single variable Q of size 100 rows and 2 columns where each row is a measurement of the world coordinates of the point. You can compute the sample mean and sample covariance of the measurements by typing (in Matlab)

```
mu = mean(Q);
C = cov(Q);
```

(a) Estimate the coordinates of the measured point on the wall.

(b) Estimate the angle of the wall relative the the X-axis of the world frame.

(c) Estimate the covariance matrix of the measurements in the robot frame; assume that the sensor is mounted so that it points in the positive X-direction of the robot.

Submit any Matlab code you might have used using the submit command:

```
submit 4421 a3 <your M-files>
```

4. **Graduate students only.** In Matlab, write a simulation for the bicycle in Question 2 that generates a graph similar to Slide 13 from Day 15. The bicycle should have a frame length of 1, and at time $t = 0$ it should be located at the world origin with wheels and frame aligned with the world x axis. Your simulation should add Gaussian distributed noise to the angles α_R and α_F and the velocity v_F at each time step. Plot the differences in the final location of the bicycle for small and large noise variances; do this for 1000 trials of your simulation.