Double-Ended Queues

CSE 2011

Winter 2011

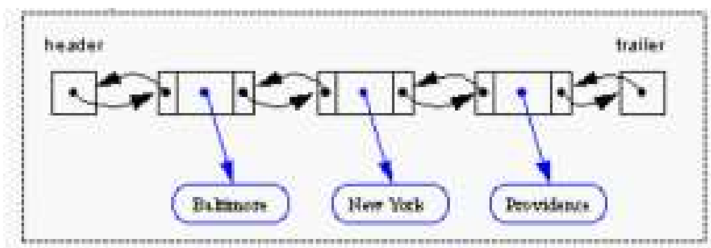
Double-Ended Queue ADT

- Deque (pronounced "deck")
- Allows insertion and deletion at both the front and the rear of the queue
- Deque ADT: operations

addFirst(*e*): insert *e* at the beginning of the deque
addLast(e): insert *e* at the end of the deque
removeFirst(): remove and return the first element
removeLast(): remove and return the last element
getFirst(): return the first element
getLast(): return the last element
isEmpty(): return true if deque is empty; false otherwise
size(): return the number of objects in the deque

Implementation Choices

- Arrays
  - Similar to queue implementation (homework)
- Linked lists: singly or doubly linked?
  - Removing at the tail costs $\theta(n)$



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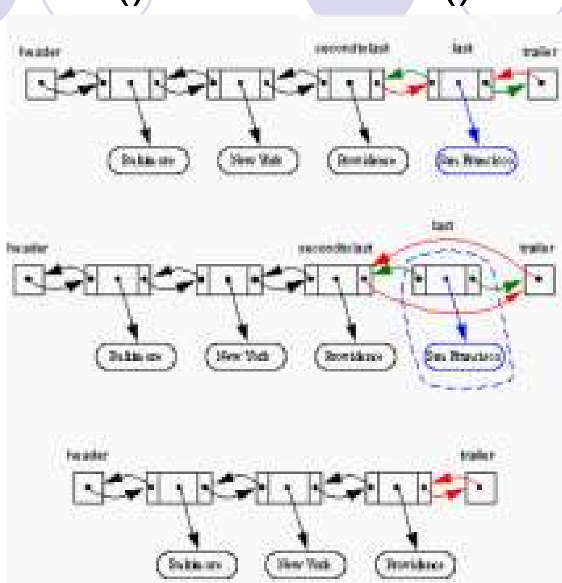
removeLast() and *addLast*()



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Implementing Stacks and Queues with Deques

Stack Method	Deque Implementation
size()	size()
isEmpty()	isEmpty()
top()	last()
push(e)	insertLast(e)
pop()	removeLast()

Queue Method	Deque Implementation
size()	size()
isEmpty()	isEmpty()
front()	first()
enqueue()	insertLast(e)
dequeue()	removeFirst()

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The Adapter Pattern

- Using methods of one class to implement methods of another class
- Example: using Deque to implement Stack and Queue

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Extendable Arrays

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Extendable Array Implementation

When *push*() is called and an overflow occurs ($n = N$):

- Allocate a new array T of capacity $2N$
- Copy contents of the original array V into the first half of the new array T
- Set $V = T$
- Perform the insertion using new array V

- Note: when the number of elements in the list goes below a threshold (e.g., $N/4$), shrink the array by half the current size N of the array.

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Time Analysis

- "*push*": inserting an element to be the last element of a list (or top of a stack)
- *add*(*e*) {
  - if (full stack) then extend the array;
  - "push" *e* to new array;

  }
- Proposition 1:

  Let *S* be a list implemented by means of an extendable array *V* as described before. The total time to perform a series of *n* "push" operations in *S*, starting from *S* being empty and *V* having size *N* = 1, is *O*(*n*).

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Pseudo-code

```
int [ ] V = new int[1]; N = 1;  top = –1;
input element e;
for( i = 0; i < n; i++ ) {
  if( stack is full ) {
    allocate a new array T of capacity 2N;
    copy V[i] to T[i] for i = 0, 1, …, N–1;      // a for loop
    set V = T;
    N = N * 2;
  }
  top = top + 1;
  V[top] = e;
  input next element e;
}
```

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Time Analysis (2)

1. All array extensions: O(?)
- Allocate a new array *T* of capacity 2*N*
- Copy *V*[*i*] to *T*[*i*] for i = 0, 1, …, *N*–1
- Set *V* = *T*

2. All "push" operations take O(*n*) (each "push" takes O(1))

Running time of all array extensions:
- If the array is extended *k* times, then $n = 2^k$
- The total number of copies is:
  $1 + 2 + 4 + 8 + \dots + 2^{k-1} = 2^k - 1 = n - 1 = O(n)$

Total = $O(n) + O(n) = O(n)$

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Increment Strategies

- java.util.ArrayList and java.util.Vector use extendable arrays.
- *capacityIncrement* determines how the array grows:
  capacityIncrement = 0: array size doubles
  capacityIncrement = *c* > 0: array adds *c* new cells
- Proposition 2:
  If we create an initially empty java.util.Vector object with a fixed positive *capacityIncrement* value, then performing a series of n push operations on this vector takes $\Omega(n^2)$ time.
- $\Omega(n^2)$: takes at least time n^2

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Increment Strategies (2)

1. Array extensions: O(?)

- Let a be the initial size of array V
- Let *capacityIncrement* = c
- If the array is extended k times then $n = a + ck$
- The total number of copies is:
  $(a) + (a+c) + (a+2c) + \ldots + (a+(k-1)c) =$
  $ak + c(1+2+\ldots+(k-1)) = ak + ck(k-1)/2 = \theta(k^2) = \theta(n^2)$
- We infer $\Omega(n^2)$ from $\theta(n^2)$

2. <u>All</u> "push" operations take O(n) (each "push" takes O(1))

Which is the better increment strategy?

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Next time ...

- Trees (chapter 7)
- Assignment 1
  - We discuss solutions in class.
  - Solutions will not be posted.

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