Merge Sort (11.1)

CSE 2011
Winter 2011

Goals

- Divide-and-conquer approach
- Solving recurrences
- One more sorting algorithm

Merge Sort: Main Idea

Based on divide-and-conquer strategy

- Divide the list into two smaller lists of about equal sizes.
- Sort each smaller list *recursively.*
- Merge the two sorted lists to get one sorted list.

Questions:

- How do we divide the list? How much time needed?
- How do we merge the two sorted lists? How much time needed?

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Dividing

- If the input list is an array A[0..N-1]: dividing takes O(1) time:
  - Represent a sub-array by two integers *left* and *right*.
  - To divide A[*left* .. *right*], compute *center*=(*left*+*right*)/2 and obtain A[*left* .. *center*] and A[*center*+1 .. *right*]
- If the input list is a linked list, dividing takes $\Theta(N)$ time:
  - Scan the linked list, stop at the $\lfloor N/2 \rfloor^{th}$ entry and cut the link.

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Merge Sort: Algorithm

- Divide-and-conquer strategy
  - recursively sort the first half and the second half
  - merge the two sorted halves together

```
void mergesort(int & A[], int left, int right)
{
        If (left < right ) {
                int center = (left + right) /2;
                mergesort(A, left, center);
                mergesort(A, center+1, right);
                merge(A, left, center+1, right);
}
```

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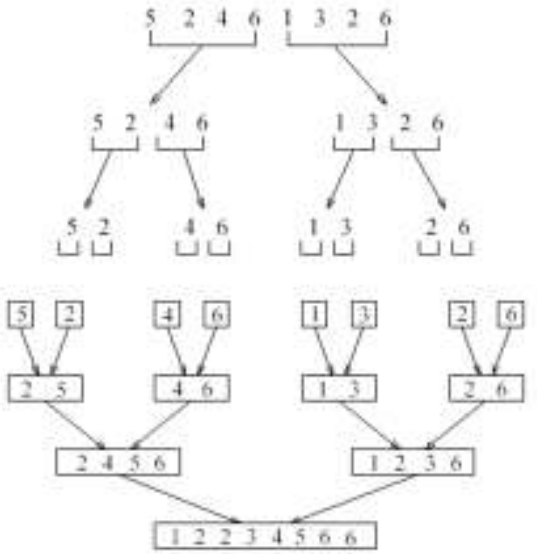

see applet http://www.cosc.canterbury.ac.nz/people/mukundan/dsal/MSort.html

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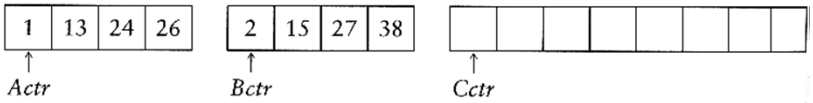
Merging

- Input: two sorted array A and B
- Output: an output sorted array C
- Three counters: *Actr*, *Bctr*, and *Cctr*
  - initially set to the beginning of their respective arrays



  - The smaller of A[*Actr*] and B[*Bctr*] is copied to the next entry in C, and the appropriate counters are advanced
  - When either input list is exhausted, the remainder of the other list is copied to C.

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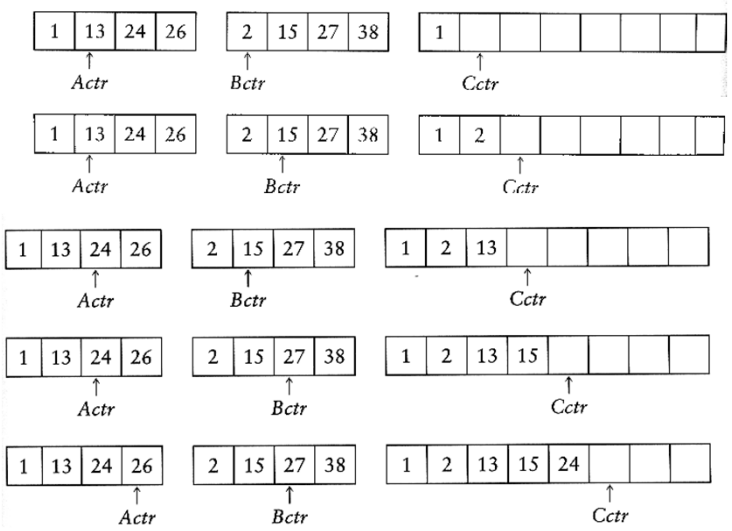
Merge: Example



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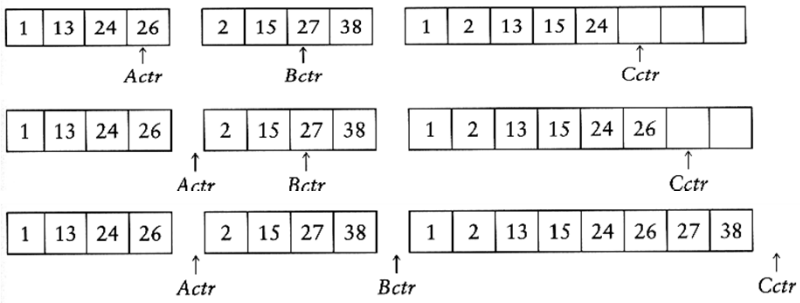
Example: Merge (2)



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Merge: Java Code

```
/**
   * Internal method that merges two sorted halves
     of a subarray.
   * @param a an array of Comparable items.
   * @param tmpArray an array to place the merged
     result.
   * @param leftPos the left-most index of the
     subarray.
   * @param rightPos the index of the start of the
     second half.
   * @param rightEnd the right-most index of the
     subarray.
   */
   private static <AnyType extends
                   Comparable<? super AnyType>>
   void merge( AnyType [ ] a, AnyType [ ] tmpArray,
     int leftPos, int rightPos, int rightEnd )
   {
      int leftEnd = rightPos - 1;
      int tmpPos = leftPos;
      int numElements = rightEnd - leftPos + 1;

// Main loop
      while( leftPos <= leftEnd && rightPos <=
      rightEnd )
         if( a[ leftPos ].compareTo( a[ rightPos ] ) <= 0
      )
            tmpArray[ tmpPos++ ] = a[ leftPos++ ];
         else
            tmpArray[ tmpPos++ ] = a[ rightPos++ ];

      while( leftPos <= leftEnd )   // Copy rest of 1st half
         tmpArray[ tmpPos++ ] = a[ leftPos++ ];

      while( rightPos <= rightEnd )
                              // Copy rest of right half
         tmpArray[ tmpPos++ ] = a[ rightPos++ ];

      // Copy tmpArray back
      for( int i = 0; i < numElements; i++, rightEnd-- )
         a[ rightEnd ] = tmpArray[ rightEnd ];
   }
```

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Merge: Analysis

- Running time analysis:
  - Merge takes $O(m_1 + m_2)$, where m_1 and m_2 are the sizes of the two sub-arrays.
- Space requirement:
  - merging two sorted lists requires linear extra memory
  - additional work to copy to the temporary array and back

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Analysis of Merge Sort

- Let T(N) denote the worst-case running time of ***mergesort*** to sort N numbers.
- Assume that N is a power of 2.
- Divide step: O(1) time
- Conquer step: 2 x T(N/2) time
- Combine step: O(N) time
- Recurrence equation:

  T(1) = 1

  T(N) = 2T(N/2) + N

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Solving the Recurrence

$$T(N) = 2T(\frac{N}{2}) + N$$
$$= 2(2T(\frac{N}{4}) + \frac{N}{2}) + N$$
$$= 4T(\frac{N}{4}) + 2N$$
$$= 4(2T(\frac{N}{8}) + \frac{N}{4}) + 2N$$
$$= 8T(\frac{N}{8}) + 3N = \cdots$$
$$= 2^k T(\frac{N}{2^k}) + kN$$

Since $N=2^k$, we have $k=\log_2 n$

$$T(N) = 2^k T(\frac{N}{2^k}) + kN$$
$$= N + N \log N$$
$$= O(N \log N)$$

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Next time ...

- Quick Sort (11.2)

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