Depth-First Search

CSE 2011
Winter 2011

Depth-First Search (DFS)

- DFS is another popular graph search strategy
  - Idea is similar to pre-order traversal (visit node, then visit children recursively)
- DFS will continue to visit neighbors in a recursive pattern
  - Whenever we visit v from u, we recursively visit all unvisited neighbors of v. Then we backtrack (return) to u.

DFS Traversal Example

DFS Algorithm

```
Algorithm DFS(s)
1.   for each vertex v
2.        do flag[v] := false;
3.   RDFS(s);

Algorithm RDFS(v)
1.   flag[v] := true;
2.   for each neighbor w of v
3.        do if flag[w] = false
4.             then RDFS(w);
```

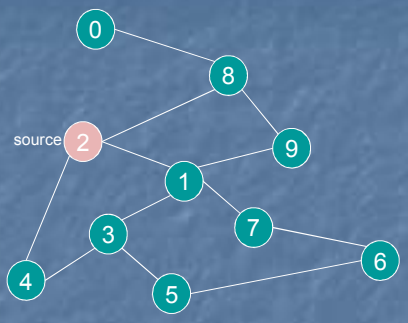
Flag all vertices as not visited

Flag v as visited; *print v*;

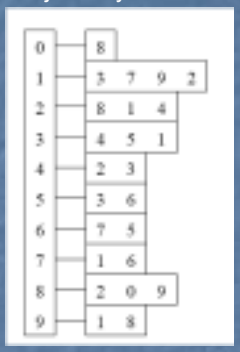
For unvisited neighbors, call *RDFS*(*w*) recursively

We can also record the paths using *prev*[ ].
Where do we insert the code for *prev*[ ]?

Example



Adjacency List



Visited Table (T/F)

0	F
1	F
2	F
3	F
4	F
5	F
6	F
7	F
8	F
9	F

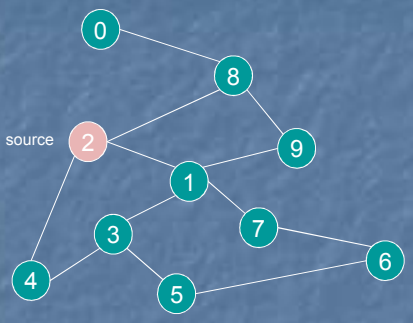
-
-
-
-
-
-
-
-
-
-

Pred

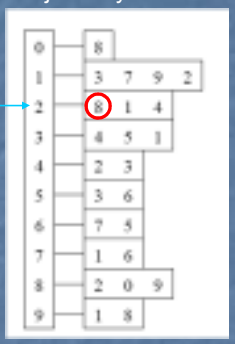
Initialize visited table (all False)

Initialize Pred to -1

5



Adjacency List



Visited Table (T/F)

0	F
1	F
2	T
3	F
4	F
5	F
6	F
7	F
8	F
9	F

-
-
-
-
-
-
-
-
-
-

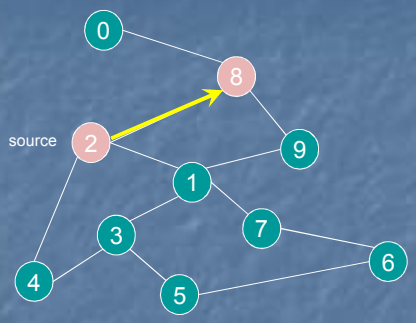
Pred

Mark 2 as visited

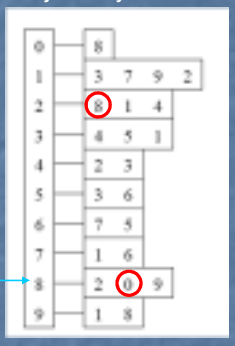
RDFS(2)

Now visit RDFS(8)

6



Adjacency List



Visited Table (T/F)

0	F	-
1	F	-
2	T	-
3	F	-
4	F	-
5	F	-
6	F	-
7	F	-
8	T	2
9	F	-

Pred

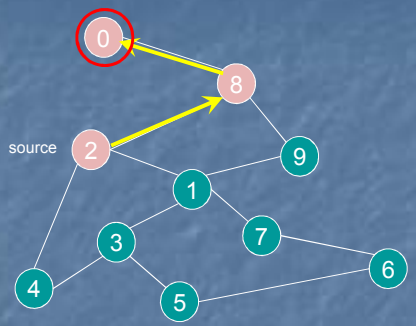
Mark 8 as visited

mark Pred[8]

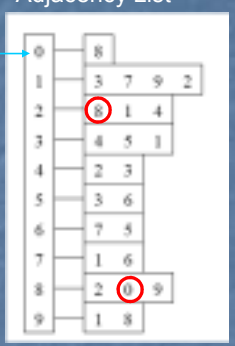
Recursive calls

RDFS(2)
RDFS(8)
2 is already visited, so visit RDFS(0)

7



Adjacency List



Visited Table (T/F)

0	T	8
1	F	-
2	T	-
3	F	-
4	F	-
5	F	-
6	F	-
7	F	-
8	T	2
9	F	-

Pred

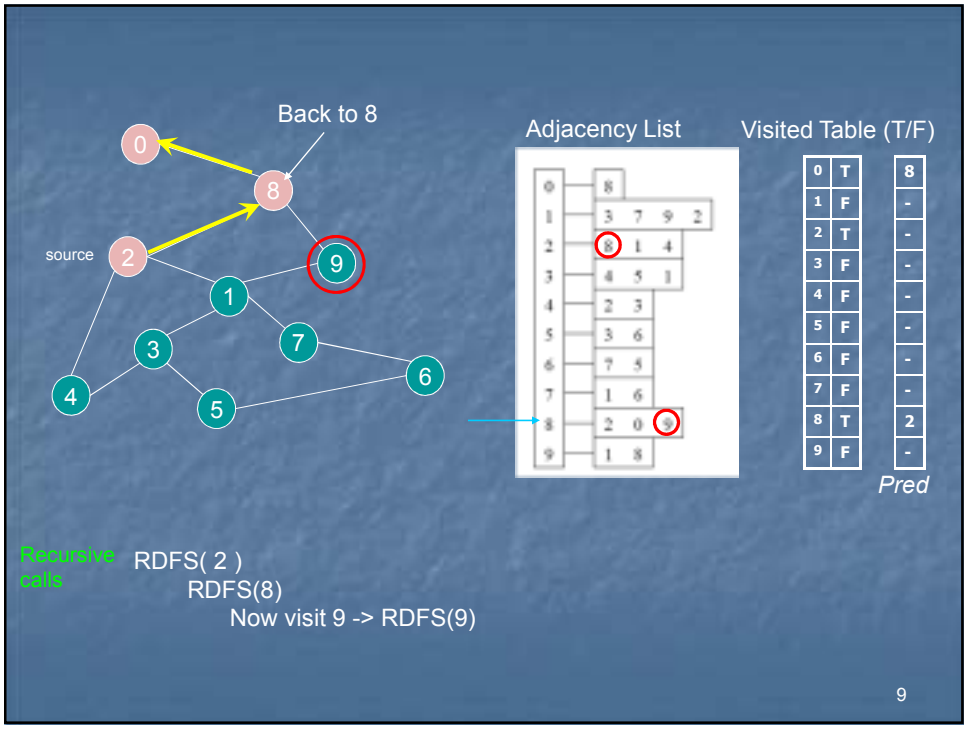
Mark 0 as visited

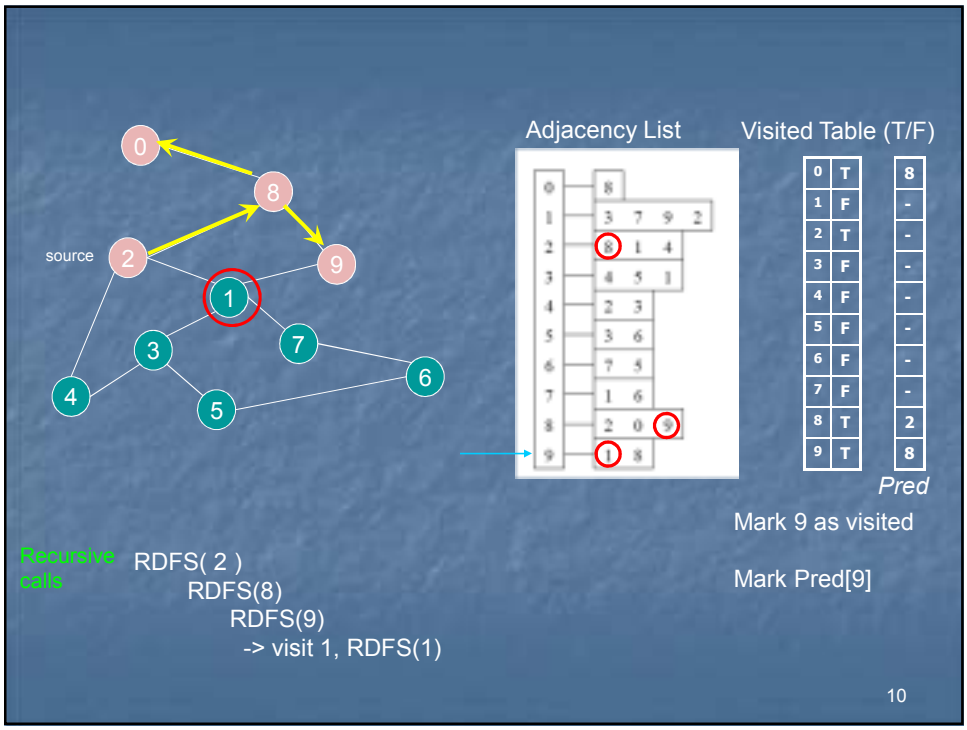
Mark Pred[0]

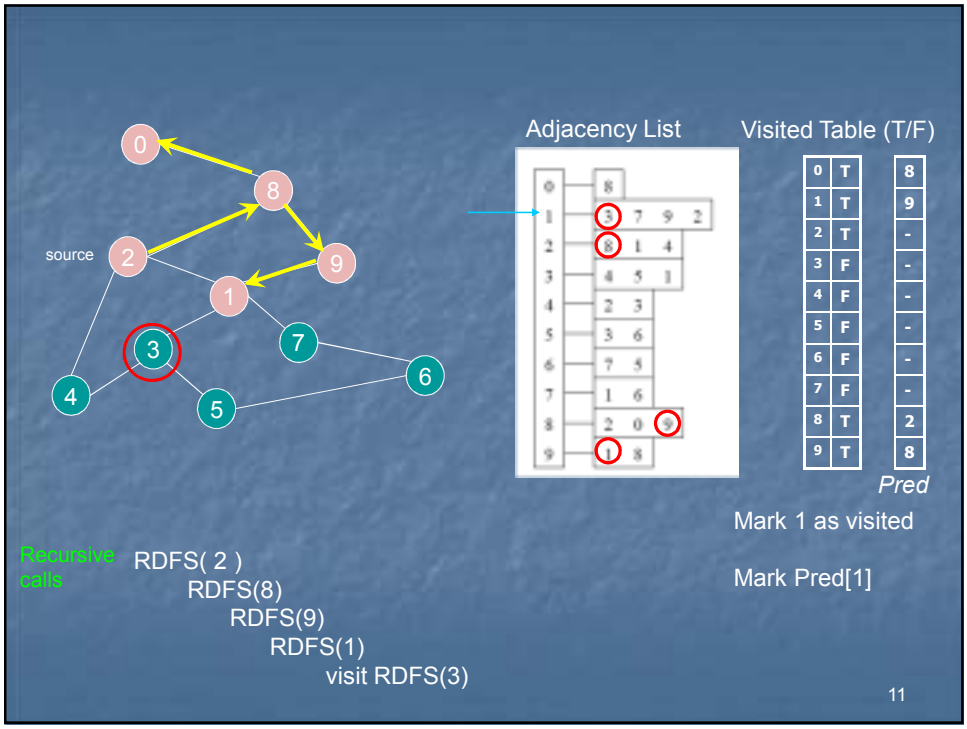
Recursive calls

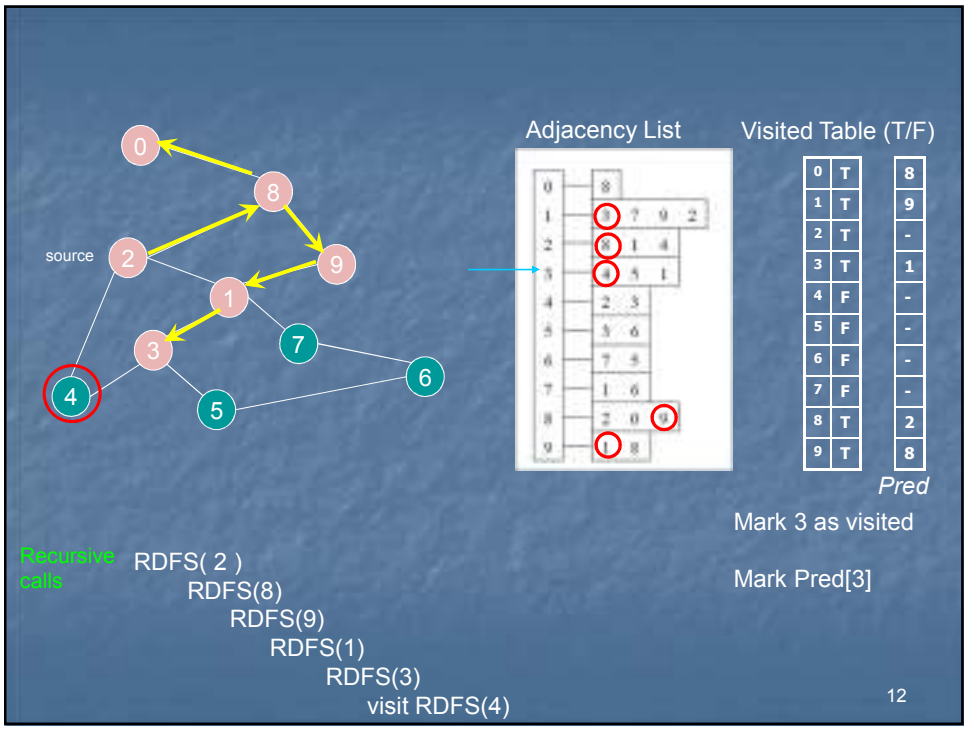
RDFS(2)
RDFS(8)
RDFS(0) -> no unvisited neighbors, return to call RDFS(8)

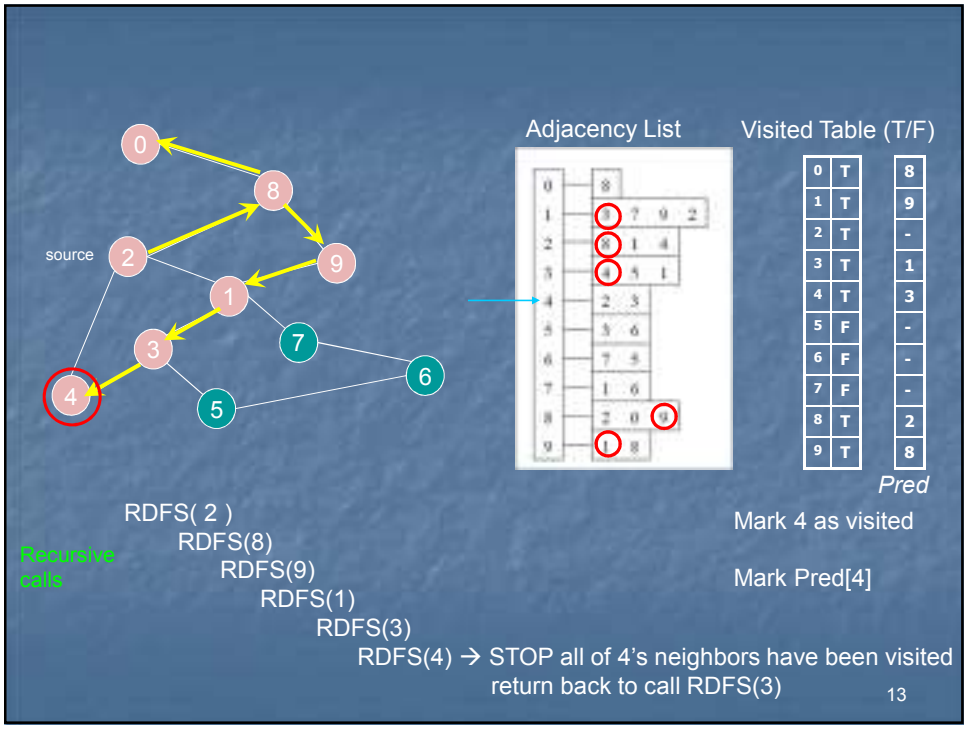
8

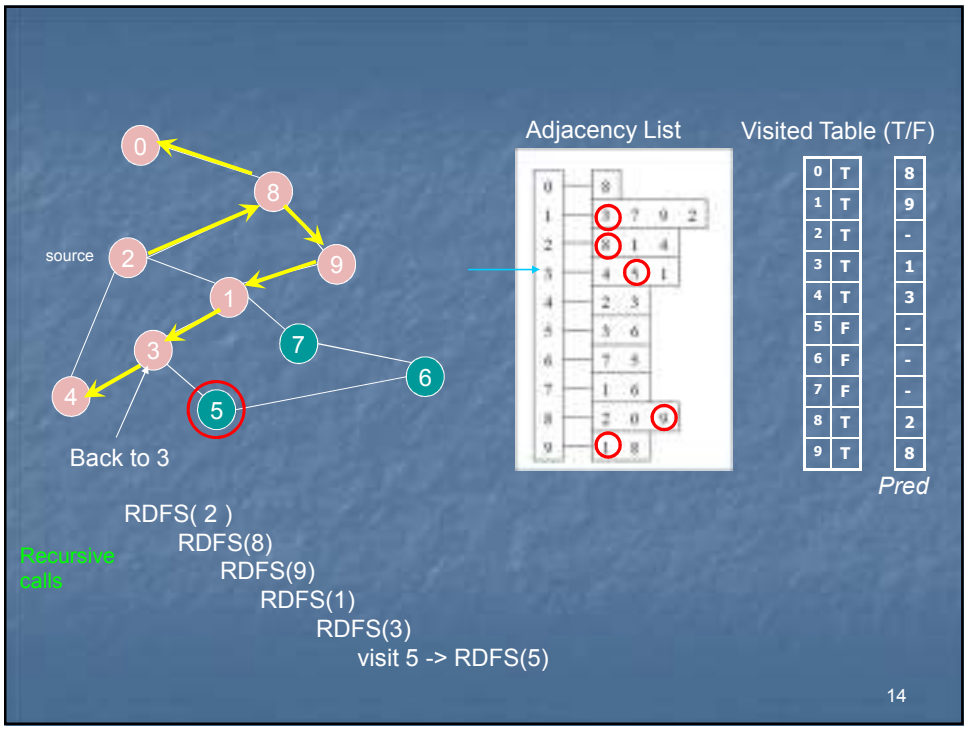

Back to 8
source
Adjacency List
Visited Table (T/F)
Pred
Recursive calls
RDFS(2)
RDFS(8)
Now visit 9 -> RDFS(9)
9

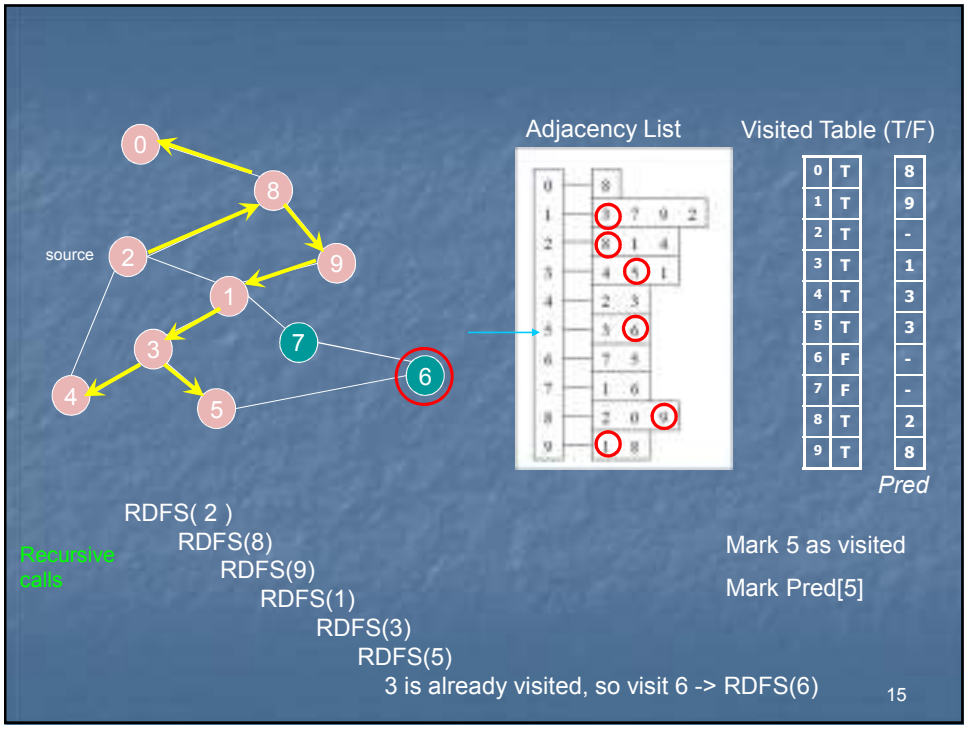

source
Adjacency List
Visited Table (T/F)
Pred
Mark 9 as visited
Mark Pred[9]
Recursive calls
RDFS(2)
RDFS(8)
RDFS(9)
-> visit 1, RDFS(1)
10

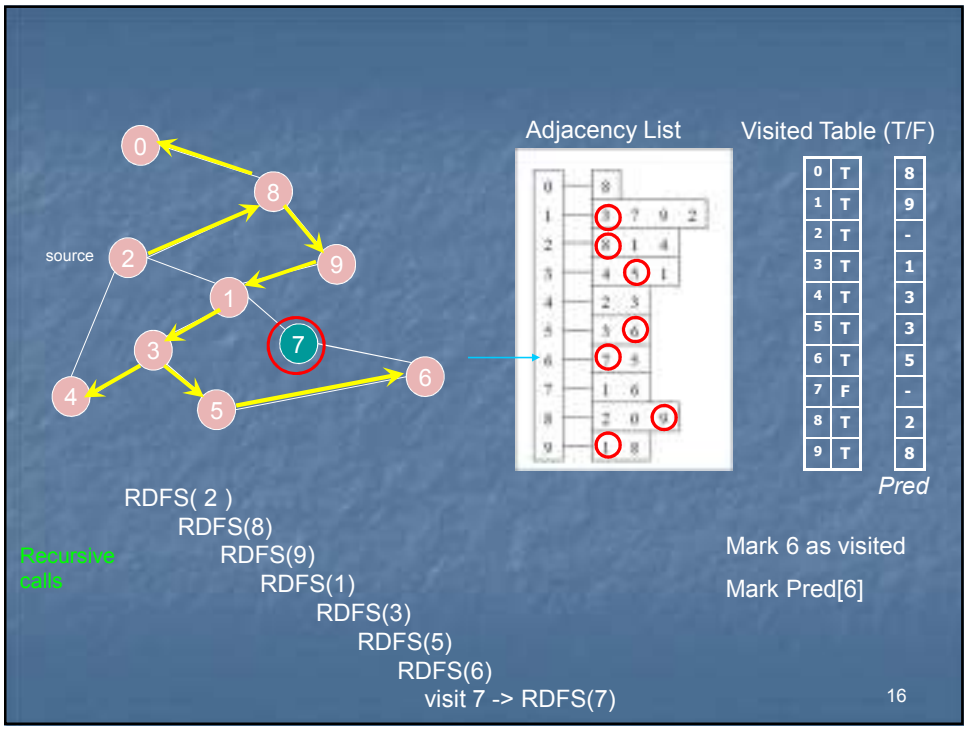

Adjacency List
Visited Table (T/F)
source
0
8
2
9
1
3
7
6
4
5
Pred
Mark 1 as visited
Mark Pred[1]
Recursive calls
RDFS(2)
RDFS(8)
RDFS(9)
RDFS(1)
visit RDFS(3)
11

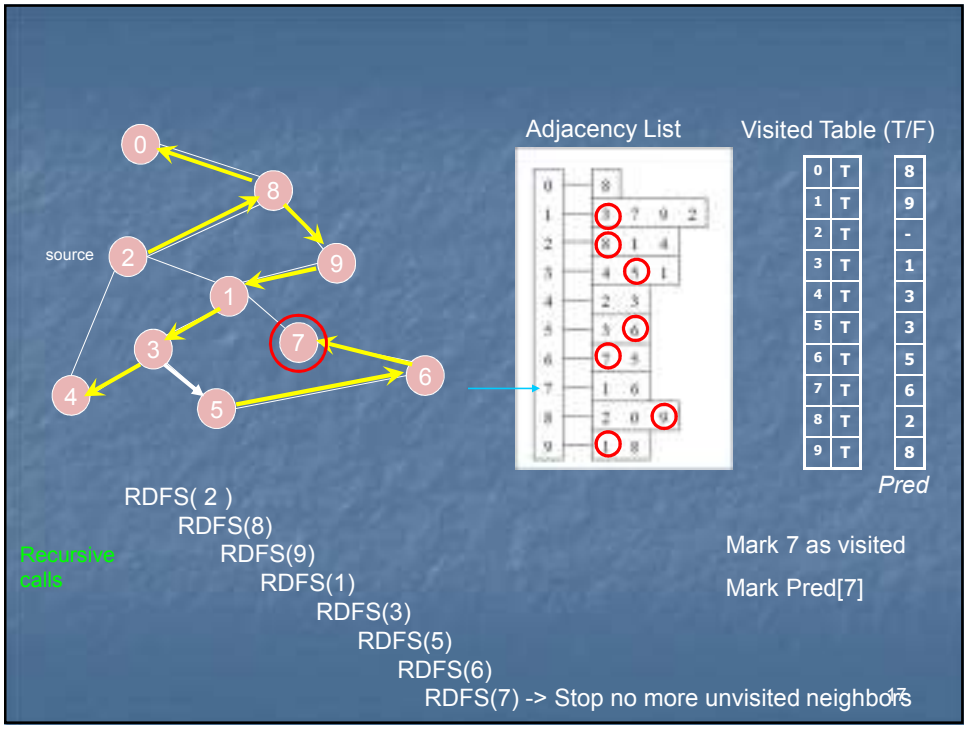

Adjacency List
Visited Table (T/F)
source
0
8
2
9
1
3
7
6
4
5
Pred
Mark 3 as visited
Mark Pred[3]
Recursive calls
RDFS(2)
RDFS(8)
RDFS(9)
RDFS(1)
RDFS(3)
visit RDFS(4)
12

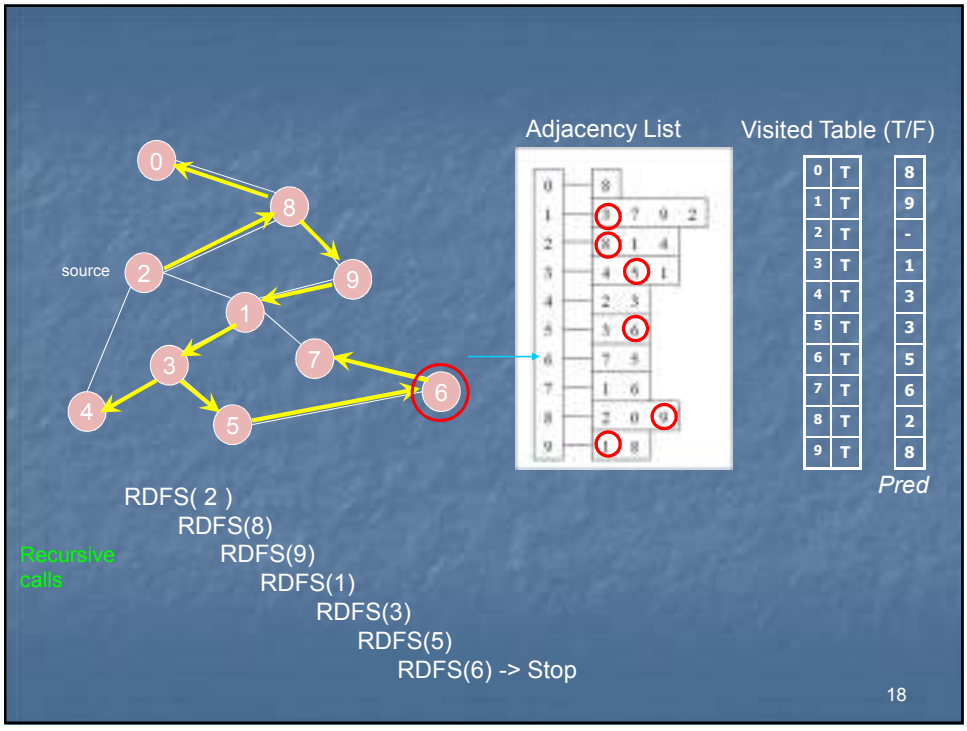

Adjacency List
Visited Table (T/F)
source
0 T
1 T
2 T
3 T
4 T
5 F
6 F
7 F
8 T
9 T
8
9
-
1
3
-
-
-
2
8
Pred
RDFS(2)
RDFS(8)
RDFS(9)
RDFS(1)
RDFS(3)
RDFS(4) → STOP all of 4's neighbors have been visited
return back to call RDFS(3)
Recursive calls
Mark 4 as visited
Mark Pred[4]
13

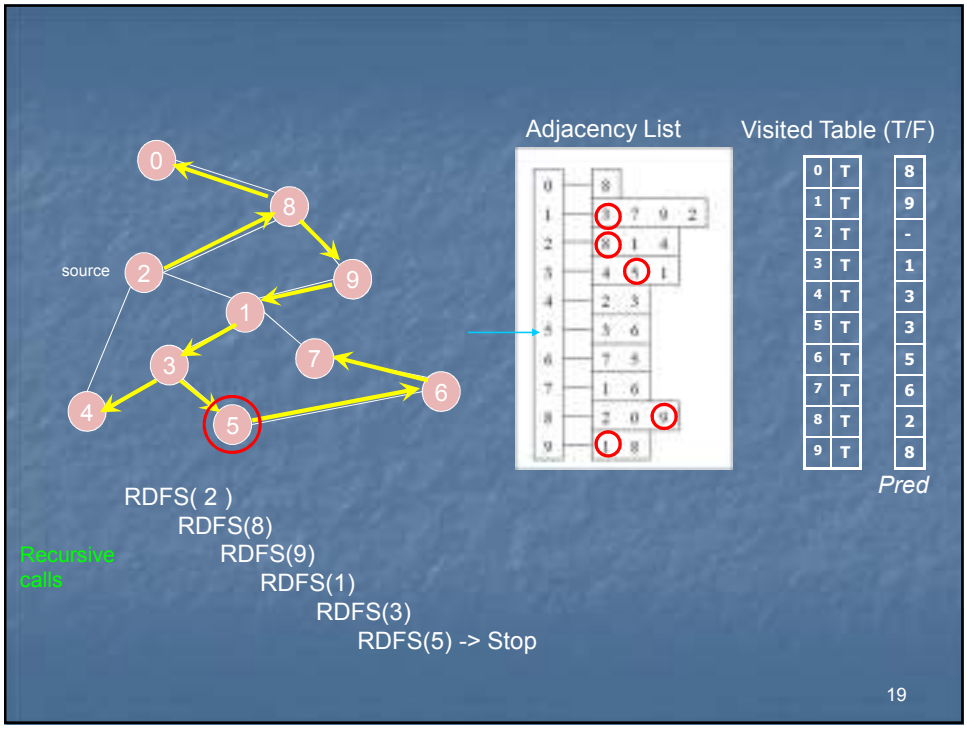

Adjacency List
Visited Table (T/F)
source
0 T
1 T
2 T
3 T
4 T
5 F
6 F
7 F
8 T
9 T
8
9
-
1
3
-
-
-
2
8
Pred
Back to 3
RDFS(2)
RDFS(8)
RDFS(9)
RDFS(1)
RDFS(3)
visit 5 -> RDFS(5)
Recursive calls
14

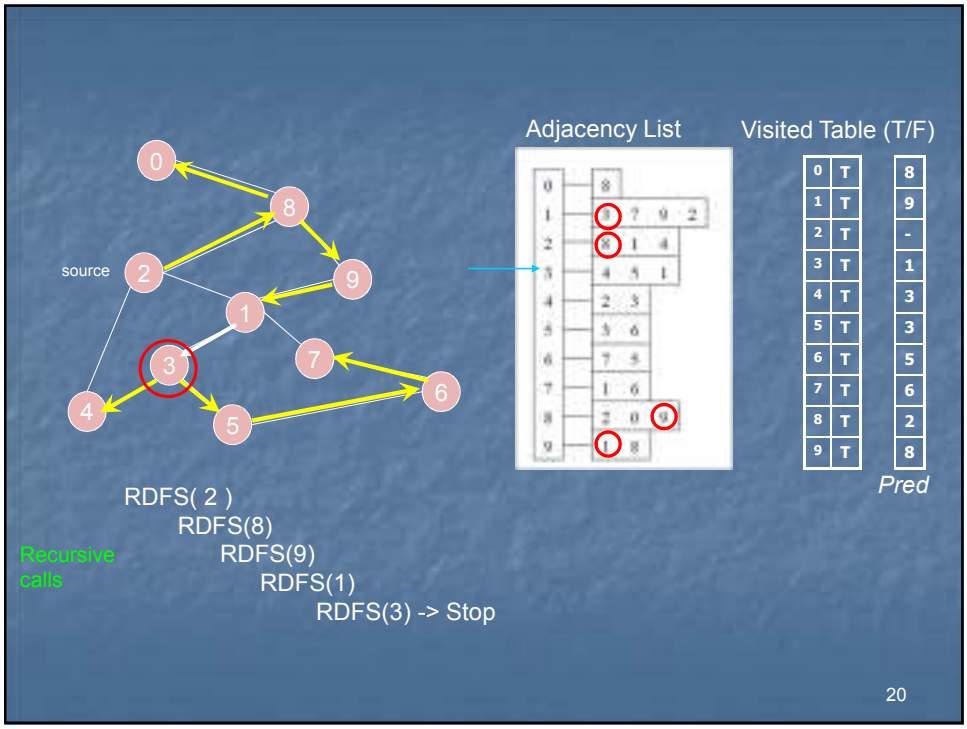

Adjacency List
Visited Table (T/F)
source
0 T
1 T
2 T
3 T
4 T
5 T
6 F
7 F
8 T
9 T
8
9
-
1
3
3
-
-
2
8
Pred
RDFS(2)
RDFS(8)
RDFS(9)
RDFS(1)
RDFS(3)
RDFS(5)
3 is already visited, so visit 6 -> RDFS(6)
Recursive calls
Mark 5 as visited
Mark Pred[5]
15

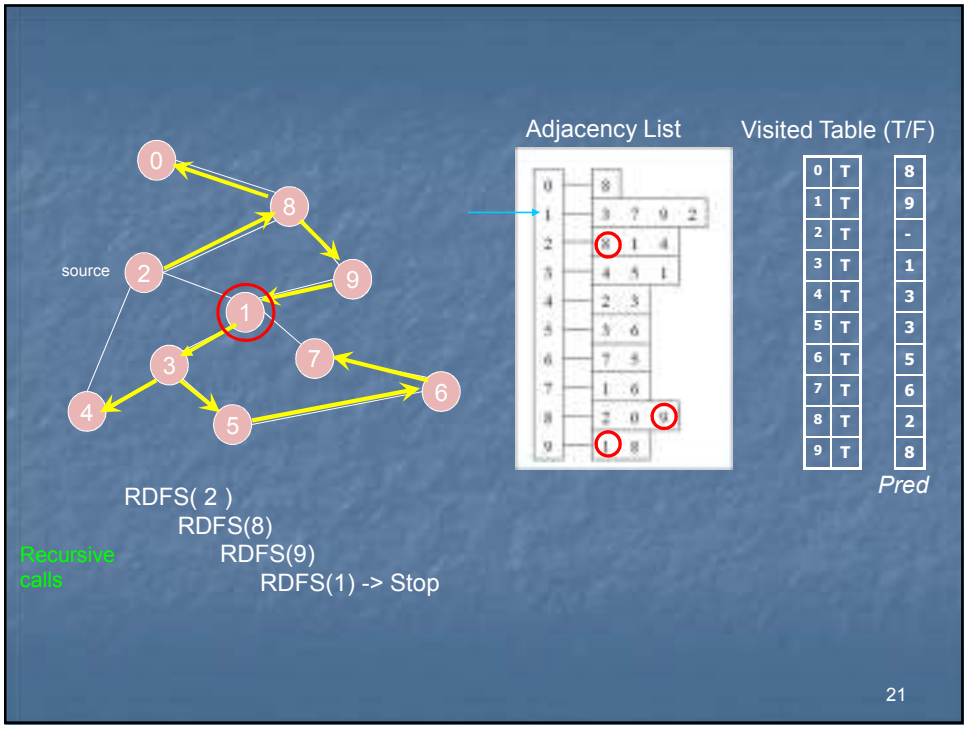

Adjacency List
Visited Table (T/F)
source
0 T
1 T
2 T
3 T
4 T
5 T
6 T
7 F
8 T
9 T
8
9
-
1
3
3
5
-
2
8
Pred
RDFS(2)
RDFS(8)
RDFS(9)
RDFS(1)
RDFS(3)
RDFS(5)
RDFS(6)
visit 7 -> RDFS(7)
Recursive calls
Mark 6 as visited
Mark Pred[6]
16

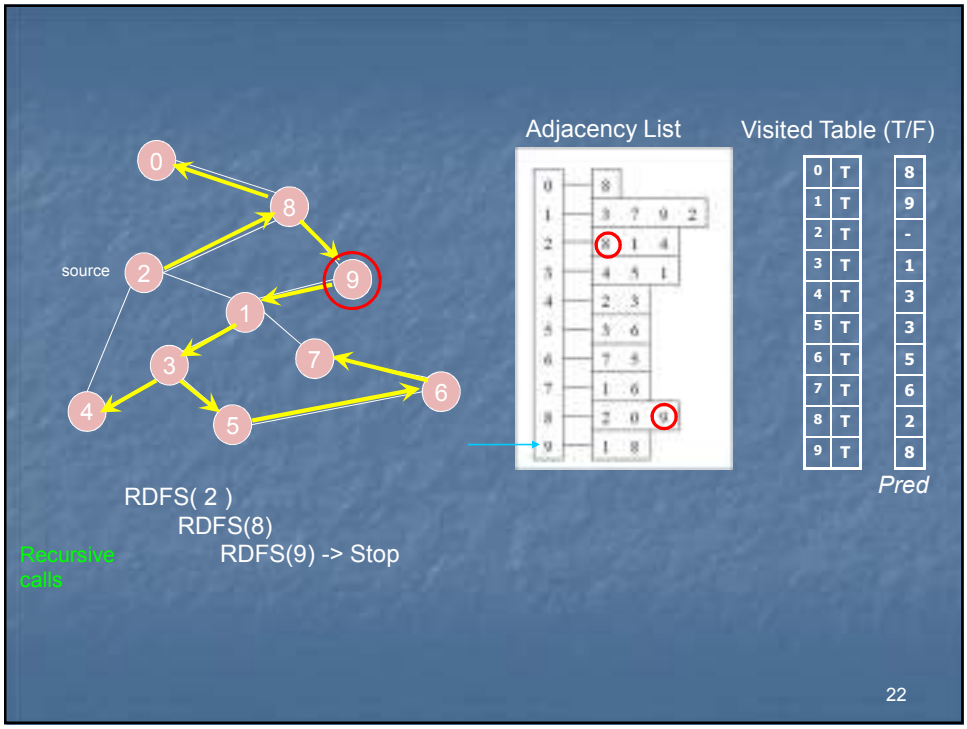

Adjacency List
Visited Table (T/F)
source
0
8
2
9
1
7
3
6
4
5
Pred
RDFS(2)
RDFS(8)
RDFS(9)
RDFS(1)
RDFS(3)
RDFS(5)
RDFS(6)
RDFS(7) -> Stop no more unvisited neighbors
Recursive calls
Mark 7 as visited
Mark Pred[7]
17

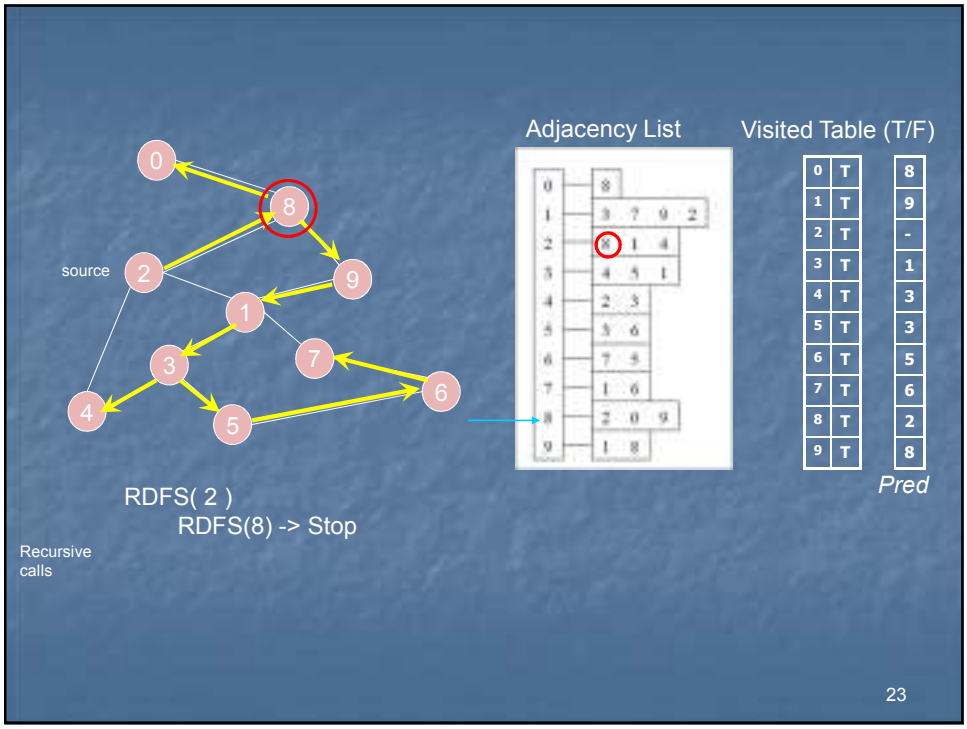

Adjacency List
Visited Table (T/F)
source
0
8
2
9
1
7
3
6
4
5
Pred
RDFS(2)
RDFS(8)
RDFS(9)
RDFS(1)
RDFS(3)
RDFS(5)
RDFS(6) -> Stop
Recursive calls
18

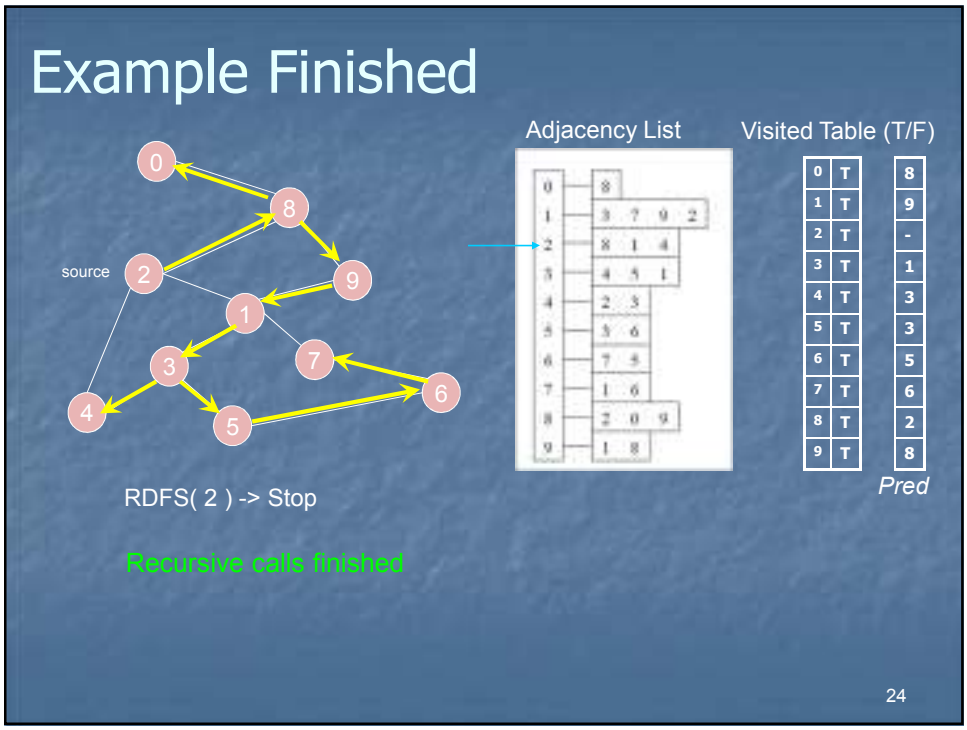

Adjacency List
Visited Table (T/F)
source
0
8
2
9
1
3
7
6
4
5
RDFS(2)
RDFS(8)
RDFS(9)
RDFS(1)
RDFS(3)
RDFS(5) -> Stop
Recursive calls
Pred
19


Adjacency List
Visited Table (T/F)
source
0
8
2
9
1
3
7
6
4
5
RDFS(2)
RDFS(8)
RDFS(9)
RDFS(1)
RDFS(3) -> Stop
Recursive calls
Pred
20


Adjacency List
Visited Table (T/F)
source
0
8
2
9
1
3
7
6
4
5
RDFS(2)
RDFS(8)
RDFS(9)
RDFS(1) -> Stop
Recursive calls
Pred
21


Adjacency List
Visited Table (T/F)
source
0
8
2
9
1
3
7
6
4
5
RDFS(2)
RDFS(8)
RDFS(9) -> Stop
Recursive calls
Pred
22


Adjacency List
Visited Table (T/F)
source
0
8
2
9
1
7
3
6
4
5
RDFS(2)
RDFS(8) -> Stop
Recursive calls
Pred
23


Example Finished
Adjacency List
Visited Table (T/F)
source
0
8
2
9
1
7
3
6
4
5
RDFS(2) -> Stop
Recursive calls finished
Pred
24

Time Complexity of DFS

- We never visited a vertex more than once.
- We had to examine the adjacency lists of all vertices.
  - $\Sigma_{\text{vertex } v} \text{degree}(v) = 2E$
- So, the running time of DFS is proportional to the number of edges and number of vertices (same as BFS)
  - O(V + E)

25

Enhanced DFS Algorithm

- What if a graph is not connected (strongly connected)?
  - Use an enhanced version of DFS, which is similar to the enhanced BFS algorithm.

```
DFSearch( G ) {
i = 1;     // component number
for every vertex v
    flag[v] = false;
for every vertex v
    if ( flag[v] == false ) {
        print ( "Component " + i++ );
        RDFS( v );
    }
}
```

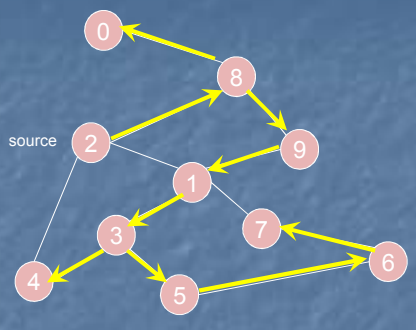
26

Applications of DFS

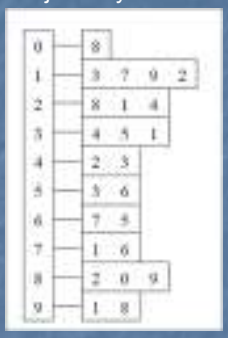
- Is there a path from source *s* to a vertex *v*?
- Is an undirected graph connected?
- Is a directed graph strongly connected?
- To output the contents (e.g., the vertices) of a graph
- To find the connected components of a graph
- To find out if a graph contains cycles and report cycles.
- To construct a DSF tree/forest from a graph

27

DFS Path Tracking



Adjacency List



Visited Table (T/F)

0	T
1	T
2	T
3	T
4	T
5	T
6	T
7	T
8	T
9	T

8
9
-
1
3
3
5
6
2
8

Pred

DFS find out path too

```
Algorithm Path(w)
1.  if pred[w] ≠ −1
2.     then
3.           Path(pred[w]);
4.  output w
```

Try some examples.
Path(0) ->
Path(6) ->
Path(7) ->

28

Next time …

- Applications of BFS and DFS
- Review

29