# Test 2

**First Name:** ______________________________

**Last Name:** ______________________________

**Student Number:** ______________________

*This test lasts* 75 minutes. *No aids allowed.*
*Make sure your test has 5 pages, including this cover page.*
*Answer in the space provided. (If you need more space, use the reverse side of the page and indicate* ***clearly*** *which part of your work should be marked.)*
*Write legibly.*
*You may use Church's Thesis to justify any of your answers.*

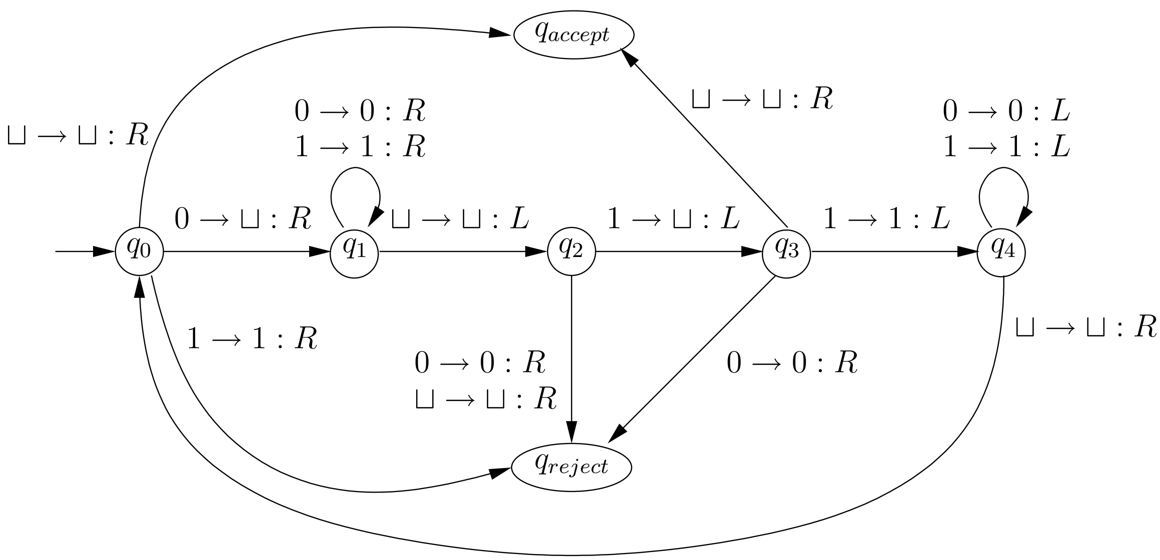
| | |
|---|---|
| Question 1 | /2 |
| Question 2 | /1 |
| Question 3 | /2 |
| Question 4 | /3 |
| Question 5 | /3 |
| Question 6 | /4 |
| Question 7 | /4 |
| Total | /19 |

**1.** [2 marks] Explain the difference between a Turing machine that recognizes a language $L$ and a Turing machine that decides a language $L$.

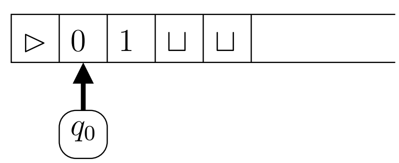
**2.** [1 marks] Give an example of a language $L$ such that $L$ is not recognizable but $\overline{L}$ is recognizable.

**3.** [2 marks] Give a high-level description of how to prove that if a language and its complement are both recognizable, then the language is decidable. Your answer must fit inside the box below. Anything written outside the box will be ignored.

**4.** [3 marks] Consider the single-tape Turing machine whose transition diagram is shown below.




If the Turing machine is initially in the configuration below (with input string 01), show the configuration of the machine after each of its first five steps. For each configuration, describe the tape contents, the head position and the state of the Turing machine.




**Bonus question** (worth 2 marks; attempt only if you have extra time): Describe exactly which strings are accepted by the Turing machine shown above.

**5.** [3 marks] Show that $\mathbb{N} \times \{A, B\}$ is countable.

**6.** [4 marks] Let $L_6 = \{\langle M \rangle : M$ is a Turing machine and for some $n \in \mathbb{N}, M$ accepts $0^n\}$. Give a brief description of an algorithm that *recognizes* $L_6$. (You do not have to prove your answer is correct.)

**7.** [4 marks] In this question, we consider Turing machines with input alphabet $\Sigma = \{0, 1\}$. Let $L = \{\langle M_1, M_2 \rangle : M_1$ and $M_2$ are Turing machines and $L(M_1) \cup L(M_2) = \{0, 1\}^*\}$. Prove that $L$ is not decidable.