Test 1

First Name: _____

Last Name: _____

Student Number: _____

This test lasts 75 minutes. *No aids allowed.*
Make sure your test has 5 pages, including this cover page.
Answer in the space provided. (If you need more space, use the reverse side of the page and indicate ***clearly*** *which part of your work should be marked.)*
Write legibly.

Question 1	/3
Question 2	/4
Question 3	/3
Question 4	/3
Question 5	/4
Question 6	/3
Total	/20

1. [3 marks] Give a high-level description of how to prove that the regular languages are closed under reversal. (I.e., how to prove that if language L is regular, then L^R must also be regular.) Your answer must fit inside the box below. Anything written outside the box will be ignored.

2. [4 marks] Let $L_2 = \{w \in \{0,1\}^* : |w| \text{ is even and } w \text{ contains an odd number of 0's}\}$. Draw a deterministic finite automaton for L_2. (You do *not* have to prove your answer is correct.)

3. [3 marks] Let $L_3 = \{w \in \{0,1\}^* :$ the first character and last character of w are the same$\}$. For example, 001010 is in L_3 but 1010 is not in L_3. Give a regular expression for L_3. (You do *not* have to prove your answer is correct.)

4. [3 marks] Give a *brief* high-level description of an algorithm which, given a finite automaton, determines whether the automaton accepts at least one string *and* rejects at least one string. If you use any of the algorithms discussed in class as subroutines, you do not have to explain how those subroutines work.

5. [4 marks] Let L_5 be the set of all binary strings that contain an equal number of 0's and 1's. For example, 00001111 and 01010101 are in L_5 but 0001111 and 110 are not in L_5. Prove that L_5 is not regular.

6. [3 marks] Suppose L is any regular language that is decided by a DFA with k states. Prove that if L is non-empty, then L contains some string of length at most k.