YORK UNIVERSITY

FACULTY OF SCIENCE AND ENGINEERING

2010 FALL TERM EXAMINATION

Course Number: CSE2001

Title: Introduction to Theory of Computation

Duration: 3 hours

No aids allowed.

- There should be 10 pages in the exam, including this page.
- Write all answers on the examination paper. If your answer does not fit in the space provided, you can continue your answer on the back of a page or on page 10, indicating clearly that you have done so.
- You may use Church's Thesis in your answer to any question.
- Write legibly.

Name ____

(Please underline your family name.)

Student Number __________________________________

1. ________________/6
2. ________________/5
3. ________________/2
4. ________________/9
5. ________________/3
6. ________________/3
7. ________________/3
8. ________________/7
9. ________________/4
10. ________________/4
11. ________________/4

Total: ________________/50

1. [6 marks] For each of the following languages, you must determine whether the language is regular, context-free, decidable, recognizable or not recognizable. For each language, circle the *leftmost* correct answer. For example, if a language is both recognizable and decidable, but not context-free, circle decidable.

(a) $\{0^n 1^n 2^n : n \in \mathbb{N}\}$.

regular context-free decidable recognizable not recognizable

(b) $\{0^n 1^n : n \in \mathbb{N}\}$.

regular context-free decidable recognizable not recognizable

(c) $\{0^n : n \in \mathbb{N}\}$.

regular context-free decidable recognizable not recognizable

(d) $\{\langle M \rangle : M$ is a Turing machine that accepts $\varepsilon\}$.

regular context-free decidable recognizable not recognizable

(e) $\{\langle M \rangle : M$ is a Turing machine and there is some input string that causes M to run forever$\}$.

regular context-free decidable recognizable not recognizable

(f) $\{\langle M, w \rangle : M$ is a Turing machine and w is a string and M halts on input $w\}$.

regular context-free decidable recognizable not recognizable

2. [5 marks] Let L and L' be languages.

[3] **(a)** Give a careful definition of what $L \leq_m L'$ means.

[2] **(b)** We proved the following statement in class: If $L \leq_m L'$ and L' is decidable, then L is also decidable. Give an outline of the proof of this statement.

3. [2 marks] Suppose you would like to prove that some language L_3 is recognizable but not decidable. You could do this by proving *two* of the following statements. Which two?

1. $\overline{L_3}$ is recognizable
2. $HALT_{TM} \leq_m L_3$
3. $EQ_{TM} \leq_m L_3$
4. $L_3 \leq_m EQ_{TM}$
5. $L_3 \leq_m A_{TM}$

Recall that $HALT_{TM} = \{\langle M, w\rangle : M$ is a Turing machine and M halts on input $w\}$ and $EQ_{TM} = \{\langle M_1, M_2\rangle : M_1$ and M_2 are Turing machines that accept exactly the same set of strings$\}$.

4. [9 marks] For each of the following statements, circle TRUE if the statement is true, or circle FALSE otherwise, and then prove that your answer is correct.

[3] **(a)** The intersection of a regular language and a decidable language is always regular.

TRUE FALSE

[3] **(b)** The union of two decidable languages is always decidable.

TRUE FALSE

[3] **(c)** The complement of a context-free language is always context-free.

TRUE FALSE

5. [3 marks] For a positive integer n, let $B(n)$ be the binary representation of n with no leading 0's. Let $L_5 = \{B(n) : n \text{ is a positive integer that is not divisible by } 3\}$. Draw the transition diagram of a deterministic finite automaton that decides L_5. You do *not* have to prove your answer is correct.

Hint: Think about how you would determine whether a number represented in decimal, like 231415123213, is divisible by 3. Then use the same algorithm, except in binary.

6. [3 marks] Give a regular expression for the set of all binary strings that do not contain 101 as a substring. You do *not* have to prove your answer is correct.

7. [3 marks] Let $L_7 = \{x : x \in \{0,1\}^* \text{ and } x = x^R \text{ and } |x| \text{ is a multiple of } 4\}$. Give a context-free grammar for the language L_7. You do *not* have to prove your answer is correct.

8. [7 marks] Let L_8 be the set of all binary strings that contain equal numbers of 0's and 1's. For example, the string 01001110 is in L_8 because it contains 4 0's and 4 1's, but the string 0011101 is not in L_8 because it contains more 1's than 0's. Consider the following context-free grammar G, which has starting symbol S.

$$S \rightarrow SS \mid 0S1 \mid 1S0 \mid \epsilon$$

[4] **(a)** Give a formal proof of the following claim.
Claim: For every $n \geq 1$, if x is any binary string that can be generated from S in n steps, then x contains equal numbers of 0's and 1's.

[1] **(b)** The claim in part (a) proves one direction of the "iff" in the following statement: The grammar G generates a string x iff $x \in L_8$. Which direction does it prove? Circle the correct answer.

IF ONLY IF

[2] **(c)** State a claim that you could prove by induction to prove the *other* direction of the "iff".

9. [4 marks] Let $L_9 = \{x\#y : x, y \in \{0,1\}^*$ and x is a substring of $y\}$. Prove that L_9 is not regular.

10. [4 marks] If string y is the last part of a string z, y is called a suffix of z. For any language $L \subseteq \Sigma^*$, let $SUFFIX(L)$ be the set of all suffixes of strings in L. Formally,

$$SUFFIX(L) = \{y : \text{there exists an } x \in \Sigma^* \text{ such that } xy \in L\}.$$

Prove that for every recognizable language L, $SUFFIX(L)$ is also recognizable.

11. [4 marks] Let $L_{11} = \{\langle M, F\rangle : M$ is a Turing machine and F is a DFA and there is no string that is accepted by both M and $F\}$. Prove that L_{11} is not decidable.

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