Homework Assignment #6
Due: March 9, 2011 at 2:30 p.m.

1. Let L be a language over some alphabet Σ.

Let $w_1 w_2 \dots w_n$ be a string (where each $w_i \in \Sigma$). Let $A[0..n]$ be an array of boolean values. We would like to fill in A with values so that it satisfies the following claim.

Claim: For $0 \leq k \leq n$,

$$A[k] = \left\{ \begin{array}{ll} \text{true} & \text{if } w_1 \dots w_k \text{ is in } L^* \\ \text{false} & \text{otherwise} \end{array} \right\}$$

(When $k = 0$, this means $A[0]$ should be true iff $\varepsilon \in L^*$.)

(a) How would you fill in the entry $A[0]$ so that it satisfies the claim above?

(b) After $A[0]$ has been filled in, suppose you fill in the rest of the array A as follows.

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for k = 1..n
    if there is a j ∈ {0, ..., k − 1} such that A[j] = true and w_{j+1} w_{j+2} ... w_k ∈ L then
        A[k] = true
    else
        A[k] = false
end for
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Give a careful proof that doing this will ensure that the claim above is satisfied.

(c) Prove that if L is decidable then L^* must also be decidable. (You may use the Church-Turing thesis here.)

Bonus Question: Deciding EQ_{DFA} Deadline for Bonus Question (only): Monday, March 14 at 11:59p.m.

2. The York University Deterministic Finite Automaton Description Format (YUDFADF) is a way to write down a description of a deterministic finite automaton in a text file. We assume the state set of the finite automaton is $Q = \{0, 1, \ldots, n-1\}$ where $n \geq 1$ and that the starting state is 0.

The first line of a YUDFADF text file contains a single integer n, representing the number of states of the DFA. The second line contains a string containing one copy of each character of the input alphabet. You may assume that the input alphabet does not contain the space character. The third line contains a non-negative integer k (the number of accepting states) followed by k integers (in the range $0..n-1$) separated by single spaces, which give the accepting states.

The fourth line contains a non-negative integer T. Following this, there are T lines. Each of these remaining lines of the description contains three items q, a, q' separated by single spaces, where q and q' are integers with $0 \leq q \leq n-1$ and $0 \leq q' \leq n-1$, and a is one of the characters in the input alphabet. (No two of these lines should have the same q and a.) If, for some state q and input character a, there is no transition given, then the finite automaton will move to a junk state and stay there if it sees an a when in state q.

Write a Java class called EQ.java, which implements the algorithm described in Theorem 4.5 of the textbook. When the main routine of the EQ class is given two descriptions of DFAs (in YUDFADF), it should output a single line that says "equivalent" if the two DFAs recognize the same language, or "not equivalent" otherwise. Some code that you might want to use to help you do this will be posted on the course web page soon. Your code should be nicely structured and well commented. This question should be submitted electronically using the "submit a6" command.