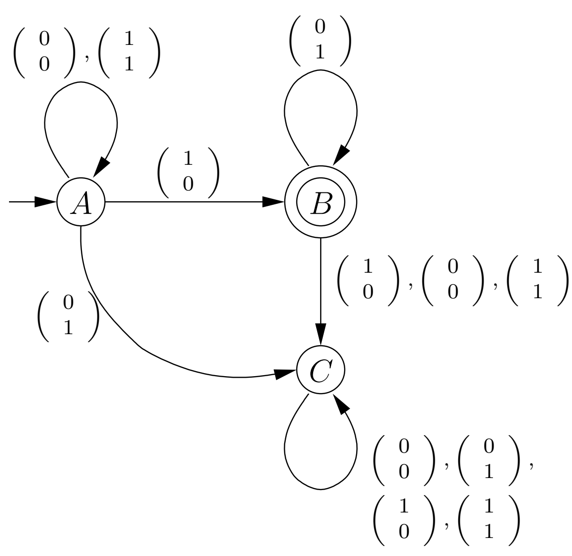
Homework Assignment #4
Due: February 16, 2011 at 2:30 p.m.

1. Consider strings over the alphabet $\Sigma = \left\{ \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \end{pmatrix} \right\}$. If w is such a string, let $t(w)$ and $b(w)$ be the numbers that are represented in binary in the top and bottom rows of w. For example, if $w = \begin{pmatrix} 1 \\ 0 \end{pmatrix}\begin{pmatrix} 1 \\ 1 \end{pmatrix}\begin{pmatrix} 0 \\ 1 \end{pmatrix}\begin{pmatrix} 1 \\ 1 \end{pmatrix}$, then $t(w) = 13$ (since 1101 is the binary representation of 13) and $b(w) = 7$ (since 0111 is the binary representation of 7). By convention, we define $t(\varepsilon) = b(\varepsilon) = 0$.

(a) Let $w, y \in \Sigma^*$. If $w = y \cdot \begin{pmatrix} 1 \\ 0 \end{pmatrix}$, explain how the values $t(w)$ and $b(w)$ can be expressed in terms of $t(y)$ and $b(y)$.

(b) Give a detailed, formal proof of the following claim about the automaton M shown below.

Claim: If w is any string in Σ^*, then
(a) M is in state A after processing w if and only if $t(w) = b(w)$, and
(b) M is in state B after processing w if and only if $t(w) = b(w) + 1$.



(c) Use part (b) to prove that the language accepted by M is $\{w \in \Sigma^* : t(w) = b(w) + 1\}$.

2. If $n \in \mathbb{N}$, let $B(n)$ be the binary string (with no leading 0's) that represents n. Now consider the language $L = \{B(n)\#B(n+1) : n \in \mathbb{N}\}$ over the alphabet $\{0, 1, \#\}$. For example, 11001#11010 is in L because 11001 is the binary representation of 25 and 11010 is the binary representation of 26, and $26 = 25 + 1$. Is L regular? Prove your answer is correct.