York University CSE2001 January 24, 2011

Homework Assignment #2
Due: February 2, 2011 at 2:30 p.m.

1.

Consider the alphabet $\left\{ \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \end{pmatrix} \right\}$. We shall use strings in this alphabet to describe two integers: one using the top row of bits and one using the bottom row. Each integer is represented in binary. For example, to represent the two integers 13 and 7 (whose binary representations are and 1101 and 111), we would use the string $\begin{pmatrix} 1 \\ 0 \end{pmatrix}\begin{pmatrix} 1 \\ 1 \end{pmatrix}\begin{pmatrix} 0 \\ 1 \end{pmatrix}\begin{pmatrix} 1 \\ 1 \end{pmatrix}$: the top row is 1101, and the bottom row is 111 (the extra 0 at the beginning of the bottom row is just padding to make the two rows the same length).

Let $PLUS2$ be the language of all strings in which the integer represented in the top row is 2 greater than the integer represented in the bottom row. For example, the string $\begin{pmatrix} 1 \\ 1 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix}\begin{pmatrix} 0 \\ 1 \end{pmatrix}\begin{pmatrix} 0 \\ 1 \end{pmatrix}\begin{pmatrix} 0 \\ 1 \end{pmatrix}\begin{pmatrix} 0 \\ 0 \end{pmatrix}$ is in $PLUS2$ because the top row represents the integer 48, the bottom row represents 46 and $48 = 46 + 2$. On the other hand, the string $\begin{pmatrix} 1 \\ 0 \end{pmatrix}\begin{pmatrix} 1 \\ 1 \end{pmatrix}\begin{pmatrix} 0 \\ 1 \end{pmatrix}\begin{pmatrix} 1 \\ 1 \end{pmatrix}$ is not in $PLUS2$ because the two rows represent 13 and 7, and $13 \neq 7 + 2$.

Draw the transition diagram of a deterministic finite automaton for the language $PLUS2$. You do not have to prove formally that your answer is correct, but you should specify, for each state of the automaton, exactly which strings can take the automaton to that state. For full credit, your automaton should be as simple as possible.

2. If L is a language over the alphabet Σ, let $\hat{L}$ be the set of all strings that can be obtained by removing exactly one character from one of the strings in L. More formally,

$$\hat{L} = \{xy : x, y \in \Sigma^* \text{ and there exists } a \in \Sigma \text{ such that } xay \in L\}.$$

(a) If $L_1 = \{011, 110\}$, what is $\hat{L_1}$?

(b) Show that, for all regular languages L, $\hat{L}$ is also regular. Your argument should have the same form as the proof of Theorem 1.47 in the textbook: first give a high-level description of your proof idea in English, then give a detailed description of the construction. In addition, you should describe, for each state of your new machine, exactly which strings will take the machine into that state (but you do not need to give a formal proof of this claim).