Homework Assignment #0
Due: January 19, 2011 at 2:30 p.m.

This assignment is based on material that is a prerequisite to this course. See the handout "Mathematical Prerequisites". After doing any rough work required, you should write your solutions on this document in the spaces provided and then hand it in. (Most of the assignments later in this course will have fewer, but longer, questions; this assignment is just intended to give you quick feedback on whether you need to review some of the background material more thoroughly.)

Name: ____

Student Number: ________________________

1. Let x be a variable that stands for a person. Let the predicate $S(x)$ represent the statement "x spends 10 hours per week solving exercises for CSE 2001". Let $P(x)$ represent the statement "x will pass the course CSE 2001". Let $R(x)$ represent the statement "x reads the textbook diligently every week."

(a) Express the following statement in English: $(S(John) \wedge R(John)) \Rightarrow P(John)$.

(b) Fill in the blank in the following to create a statement that is logically equivalent to the statement in part (a).

$\neg P(John) \Rightarrow$ ________________________________

(c) Express the following statement in English: $\forall x\ (S(x) \Rightarrow P(x))$.

(d) Write down the following English statement using the predicates defined above: Everyone who will pass CSE2001 must read the textbook diligently every week.

2. The last two digits of your student number form a decimal number between 0 and 99. Write down the binary representation of that number here:

____

3. For each of the following sets, determine whether the set is finite or infinite. If the set is finite, write down an explicit list of all the elements in the set. If the set is infinite, say so and list five elements of the set. (Assume $0 \in \mathbb{N}$.)

(a) $A = \{n \in \mathbb{N} : \exists m \in \mathbb{N}\ n + m = 3\}$.

(b) $B = \{n^2 : n \in \mathbb{N} \text{ and } n < 6\}$.

(c) $C = \{(n, m) \in \mathbb{N} \times \mathbb{N} : n = m^2\}$.

(d) $D = \{1, 7, 8\} \times \{0, 1\}$.

(e) $E = B \cap C$.

(f) $F = \mathscr{P}(\{1, 2, 3\})$

(g) $G = \mathscr{P}(\{\})$

4. Give a careful proof that $\forall n \in \mathbb{N}$, if n is even then n^2 is even.

5. Consider the function f defined by $f(x) = x^2$. If the domain of the function f is $\mathbb{Z}$, is the function a one-to-one correspondence from $\mathbb{Z}$ to the set $S = \{x^2 : x \in \mathbb{Z}\}$? Prove your answer is correct.

6. Let $A = \{x^2 + 2x : x \in \mathbb{R}\}$ and $B = \{y \in \mathbb{R} : y \geq -1\}$. Prove that $A \subseteq B$.

7. For $i \geq 1$, let A_i be a set. For $i \geq 1$, let the sets B_i and C_i be defined as follows.

$$\begin{aligned} B_1 &= A_1 \\ B_i &= B_{i-1} \cup A_i, \text{ for } i \geq 2 \\ C_1 &= \overline{A_1} \\ C_i &= C_{i-1} \cap \overline{A_i}, \text{ for } i \geq 2 \end{aligned}$$

Give a careful proof that, for all $n \geq 1, \overline{B_n} = C_n$. (Hint: You might want to use mathematical induction, but induction on what to prove what claim?)