Homework Assignment #9
Due: December 9, 2010

1. Recall the following definition of **ZPP** from class. A language L is in **ZPP** iff there is a polynomial p and a randomized TM M such that

- if $x \in L$, M outputs "yes" on input x,
- if $x \notin L$, M outputs "no" on input x, and
- for all x, the expected running time of M on input x is at most $p(|x|)$.

We can define another class **DKP** as follows. A language L is in **DKP** iff there is a polynomial p and a randomized Turing machine M that always halts within $p(|x|)$ steps on input x and outputs "yes", "no" or "I don't know" such that

- if $x \in L$, M outputs "yes" or "I don't know",
- if $x \notin L$, M outputs "no" or "I don't know", and
- for all x the probability that M outputs "I don't know" is at most $\frac{1}{2}$.

Why stop there? We define yet another class $\mathbf{ZPP}'$ as follows. A language L is in $\mathbf{ZPP}'$ iff there is a polynomial p and a randomized TM M such that

- if $x \in L$, M outputs "yes" if M terminates,
- if $x \notin L$, M outputs "no" if M terminates, and
- for all x, the expected running time of M on input x is at most $p(|x|)$.

Prove that $\mathbf{ZPP} = \mathbf{DKP} = \mathbf{ZPP}'$.