Homework Assignment #7
Due: November 16, 2010

1. A circuit is a directed graph with the following properties. Let $V = \{1, 2, \ldots, n\}$ be the vertex set of the graph. Each edge (u, v) in the graph has the property that $u < v$ (and hence there are no cycles in the graph). Each node is labelled with AND, OR, NOT or INPUT. Each NOT node has indegree 1. Each AND and OR node has indegree 2. Each INPUT node has indegree 0. Given a circuit and an assignment of a boolean value $val(u)$ to each INPUT node u, we can define the value of every other node inductively as follows.

- If u is a NOT node and (v, u) is an edge in the circuit, then $val(u) = \neg val(v)$.
- If u is an AND node and (v, u) and (w, u) are two distinct edges in the circuit, then $val(u) = val(v) \wedge val(w)$.
- If u is an OR node and (v, u) and (w, u) are two distinct edges in the circuit, then $val(u) = val(v) \vee val(w)$.

In the circuit value problem, you are given a circuit C, an assignment of boolean values to the input nodes and you must determine the value of node n.

A graph is called planar if it can be drawn on a plane without any pair of edges intersecting each other. A planar circuit is a circuit whose graph is planar.

(a) Design a planar circuit to compute XOR. I.e., design a planar circuit with two INPUT nodes such that if the two INPUT nodes are assigned the values x and y, then $val(n) = x$ XOR y.

(b) The planar circuit value problem is the same as the circuit value problem, except we insist that the input circuit is planar. We shall prove next Tuesday that the circuit value problem is **P**-complete. Show that the planar circuit value problem is also **P**-complete. (Hint: use part (a).)