Homework Assignment #4
Due: October 26, 2010

1. Let f and g be functions from $\mathbb{N}$ to $\mathbb{N}$. Define $h(n) = \min(f(n), g(n))$. Prove that $\mathbf{TIME}(f(n)) \cap \mathbf{TIME}(g(n)) = \mathbf{TIME}(h(n))$.

Are there any assumptions that you need to make about f and g in order for your proof to work? (Try to make the assumptions as weak, and as reasonable, as possible.)

2. Let f be any function from $\mathbb{N}$ to $\mathbb{N}$. Prove that $\mathbf{SPACE}(f(n))$ is closed under union. *I.e.*, prove that for all languages L_1 and L_2 in $\mathbf{SPACE}(f(n))$, $L_1 \cup L_2$ is also in $\mathbf{SPACE}(f(n))$.

Are there any assumptions that you need to make about f in order for your proof to work? (Try to make the assumptions as weak, and as reasonable, as possible.)