Homework Assignment #1
Due: September 28, 2010

1. In this question, we consider single-tape Turing machines. Suppose we modify the definition of a Turing machine so that it gets an additional power called "reverse". Thus, at each step a Turing machine can do one of four things: move the head left one square, move the head right one square, leave the head where it is, or perform a reverse operation. The reverse operation cuts the tape to the left of the current head position and to the right of the rightmost $\triangleright$ symbol, flips over the portion of the tape between those two cuts, and then glues it back together. The head does not move during a reverse operation. For example, if the head is at the c on a tape containing $\boxed{\triangleright \triangleright \mathit{abbabcdded} \sqcup \sqcup \cdots}$ a reverse operation would change the contents of the tape to $\boxed{\triangleright \triangleright \mathit{babbacdded} \sqcup \sqcup \cdots}$. A Turing machine with this additional power is called a reversing Turing machine (RTM).

(a) Describe using "Turing machine pseudocode" an RTM for deciding the language of all binary palindromes. Try to make your RTM as fast as possible. Describe the worst-case running time of your RTM using big-Θ notation.

(b) Describe how any RTM can be simulated with an ordinary Turing machine. Make your simulation as efficient (in terms of time) as you can. (But don't worry about constant factors).

(c) If an RTM runs in $T(n)$ steps (in the worst case) on inputs of size n, what is the worst-case running time of your simulation of it?