Digital Transmission of Analog Data:

PCM and Delta Modulation

Required reading:
Garcia 3.3.2 and 3.3.3

CSE 3213, Fall 2010
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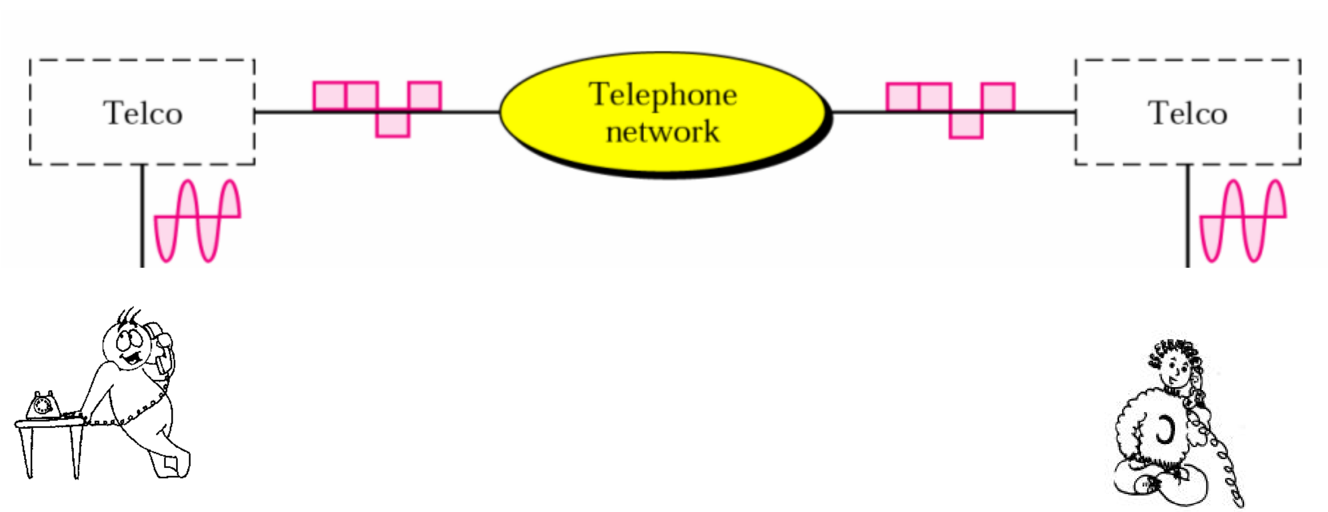
Digital Transmission of Analog Data

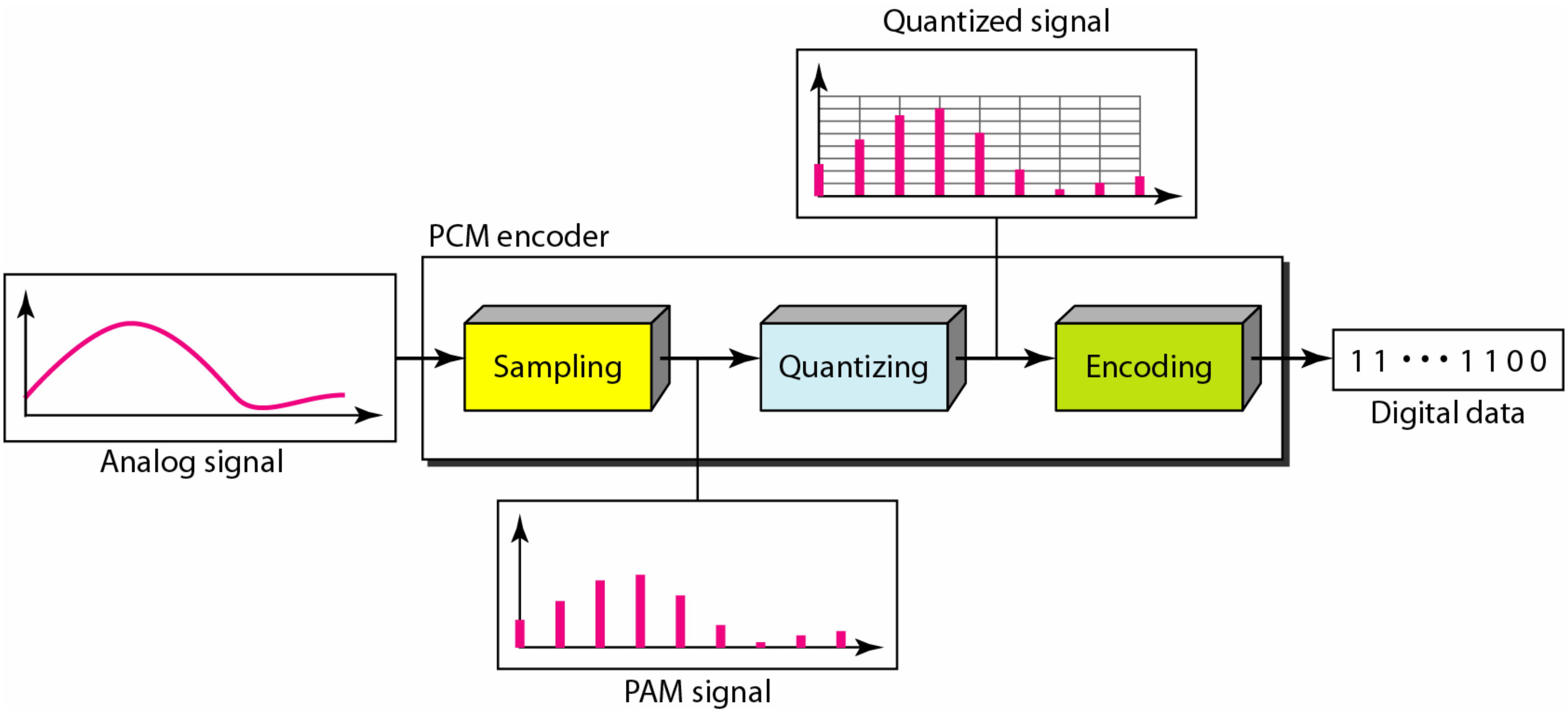
Digitization – **process of converting analog data into digital signal**

- **example: telephone system**
  - **human voice ↔ analog data ↔ analog signal ?!**
  - **analog signal is sensitive to noise, especially over long distance** (cannot be perfectly reconstructed)
  - **solution:**
    (1) digitize the analog signal at the sender
    (2) transmit digital signal
    (3) convert digital signal back to analog data at the receiver



Digital Transmission of Analog Data (cont.)

Example **[PCM procedure]**

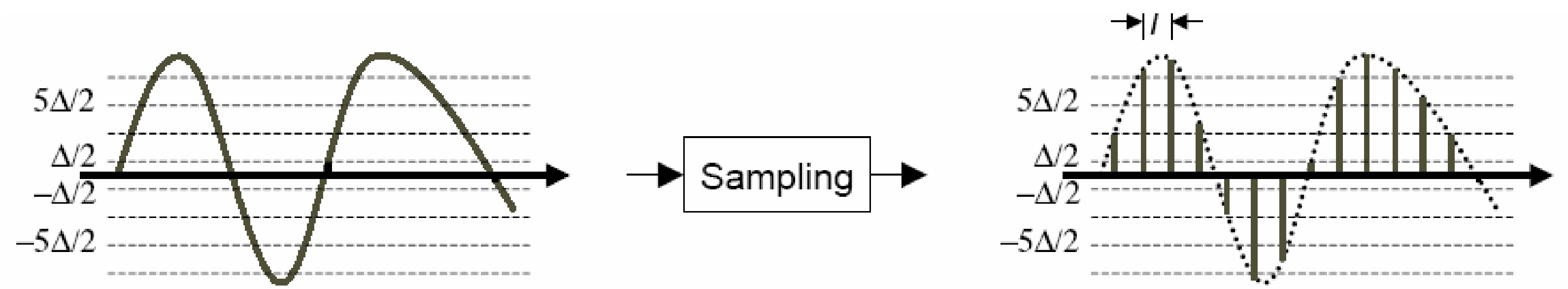


Digital Transmission of Analog Data (cont.)

Digitization Procedure – **aka Pluse Code Modulation (PCM), consists of 2 steps**

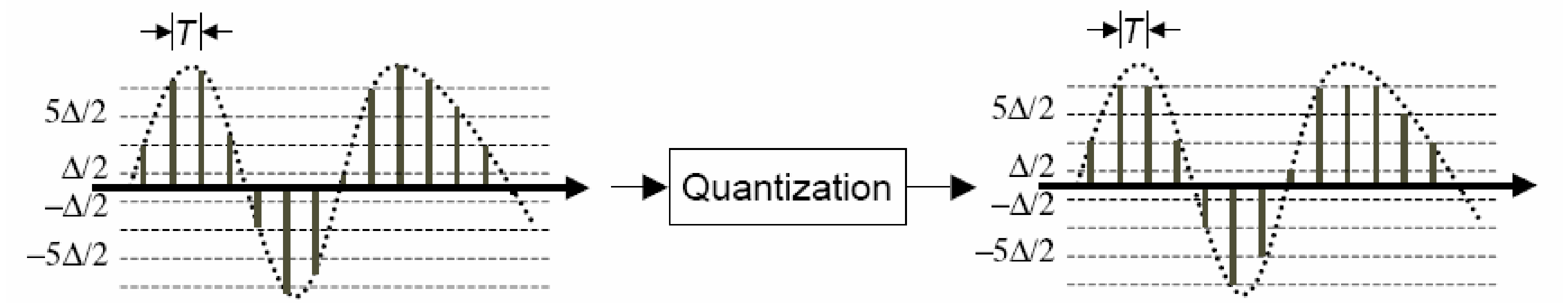
(1) sampling – obtain signal values at equal intervals (T)

(2) quantization – approximate samples to certain values



analog signal
(continuous-time, continuous-amplitude)

discrete-time signal
(discrete-time, continuous-amplitude)



discrete-time signal
(discrete-time, continuous-amplitude)

digital signal
(discrete-time, discrete-amplitude)

Sampling

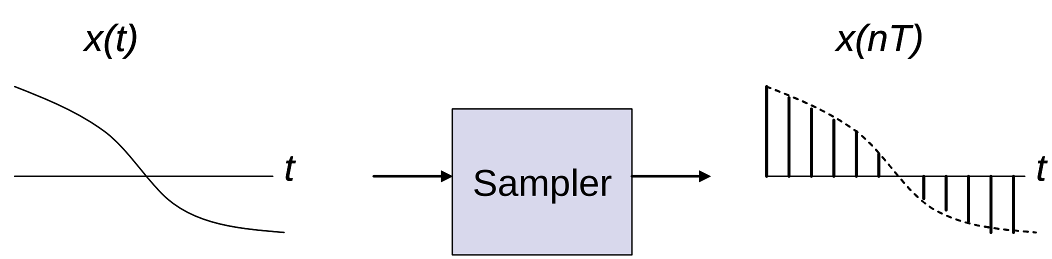
Sampling – **aka Pulse Amplitude Modulation (PAM)**

- **"digitization in time" - sampling process results in signal that is discrete in time but analog in amplitude!**
- **choice of sampling interval T is determined by how fast a signal changes, i.e. frequency content of the signal**

"*Nyquist Sampling Rate*" Theorem:

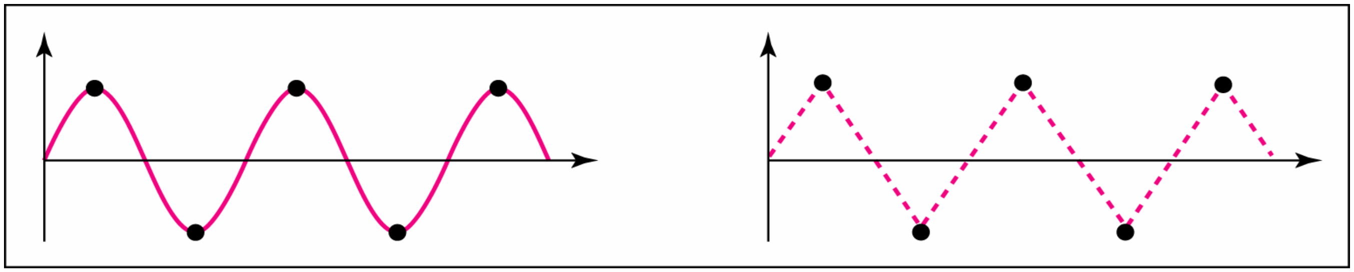
To ensure accurate reproduction of an analog signal, the sampling rate must be at least 2*(highest signal freq.).

$$\text{sampling rate} = \frac{1}{T} = 2 * \text{max\_signal\_freq}$$

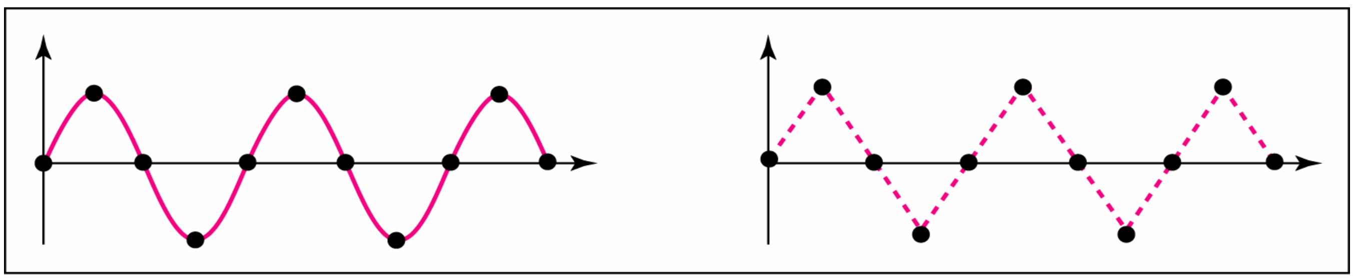


Sampling (cont.)

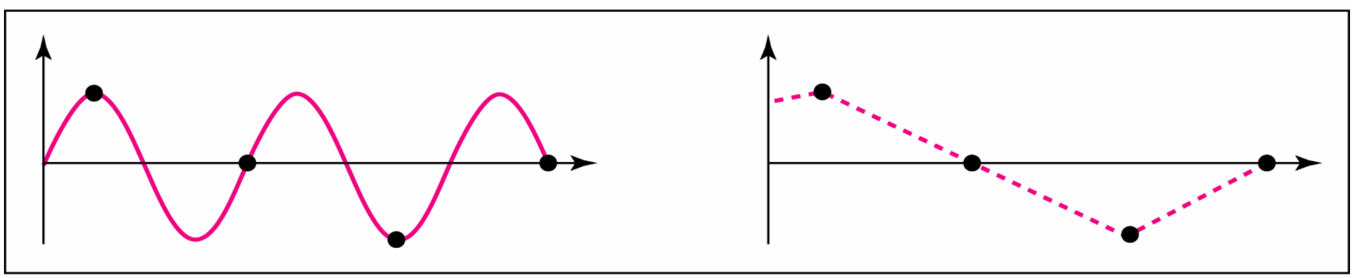
Example **[Recovery of a sampled sine wave for different sampling rates]**



a. Nyquist rate sampling: $f_s = 2\ f$



b. Oversampling: $f_s = 4\ f$



c. Undersampling: $f_s = f$

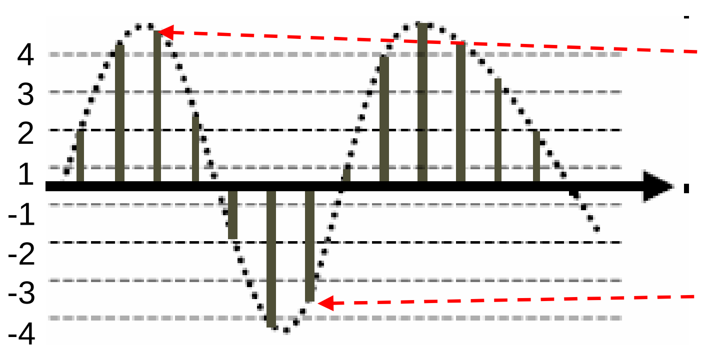
Quantization

Quantization

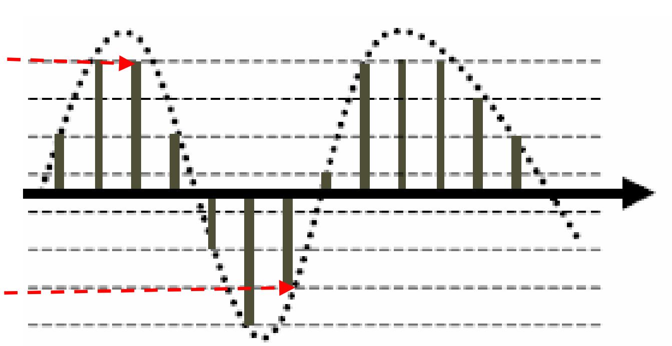
- PAM signal samples have amplitudes of '∞ precision" – direct encoding of such amplitudes would require ∞ number of bits (digital pulses) per sample
- to convert PAM signal to digital signal (that is practical for transmission), each sample has to be 'rounded up' to the nearest of M possible quantization levels

M quantization levels ⇔ m=$\log_2$(M) bits per level

- M↑ ⇒ better precision ☺, more bits per sample ☹
- M↓ ⇒ poor precision ☹, fewer bits per sample ☺



PAM signal

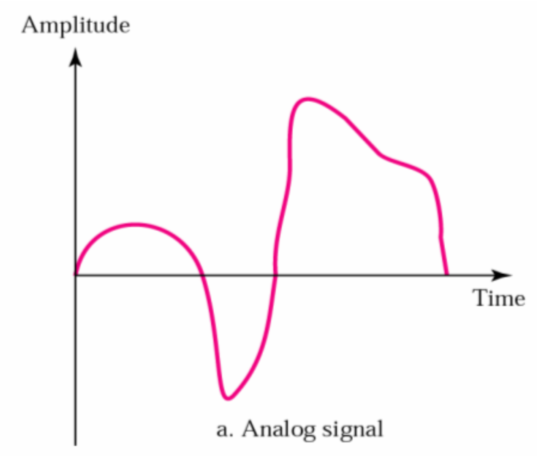


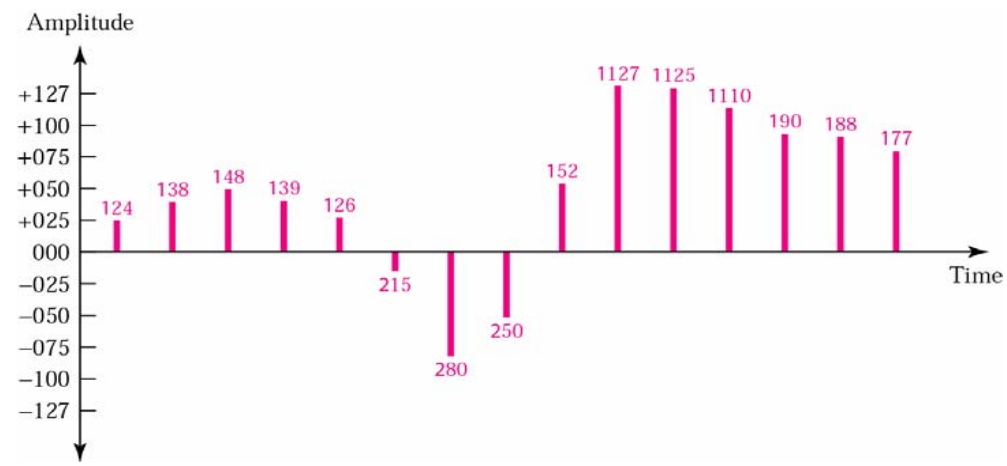
quantized signal

Quantization (cont.)

Example [Quantization of PAM Signal]

Assume an analog signal, as shown below, has to be quantized using at most 8-bits per sample.
How many different quantization levels are allowed / should be used?





+024	00011000	−015	10001111	+125	01111101
+038	00100110	−080	11010000	+110	01101110
+048	00110000	−050	10110010	+090	01011010
+039	00100111	+052	00110110	+088	01011000
+026	00011010	+127	01111111	+077	01001101

Sign bit
+ is 0 – is 1

Quantization (cont.)

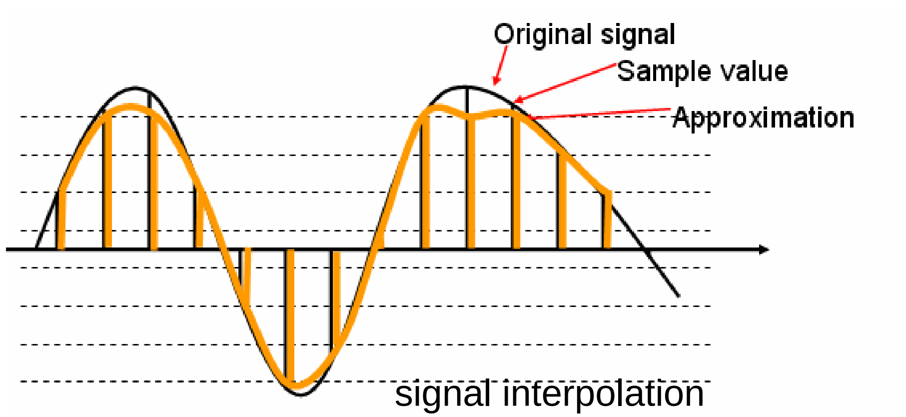
Quantization Error – **by quantizing the PAM signal, the original signal is now only approximated & cannot be 100% recovered**

- **effect known as quantizing error or quantizing noise**
- **SNR ratio due to quantizing noise can be expressed as**

bits per sample

$$\text{SNR [dB]} \approx 6m + 1.76 \text{ [dB]}$$

- **every additional bit used in quantizer will increase SNR by 6 [dB]**
  - **# of quantization levels ↑ ⇒ higher SNR ⇒ better (received) signal quality**



Quantization (cont.)

Example **[voice signal in telephone system]**

Natural human voice occupies the range of 80 – 3,400 [Hz].
Human ear can tolerate SNR of 40 [dB].

Assume we want to transmit human voice in digitized form. What bit rate [bps] should be supported by the channel to enable such transmission?

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(1) Sampling rate?!

Based on Nyquist Sampling Theorem:

max freq. = 4 [kHz] $\Rightarrow$ sampling rate = 2*4 [kHz] = 8000 [samples/sec]

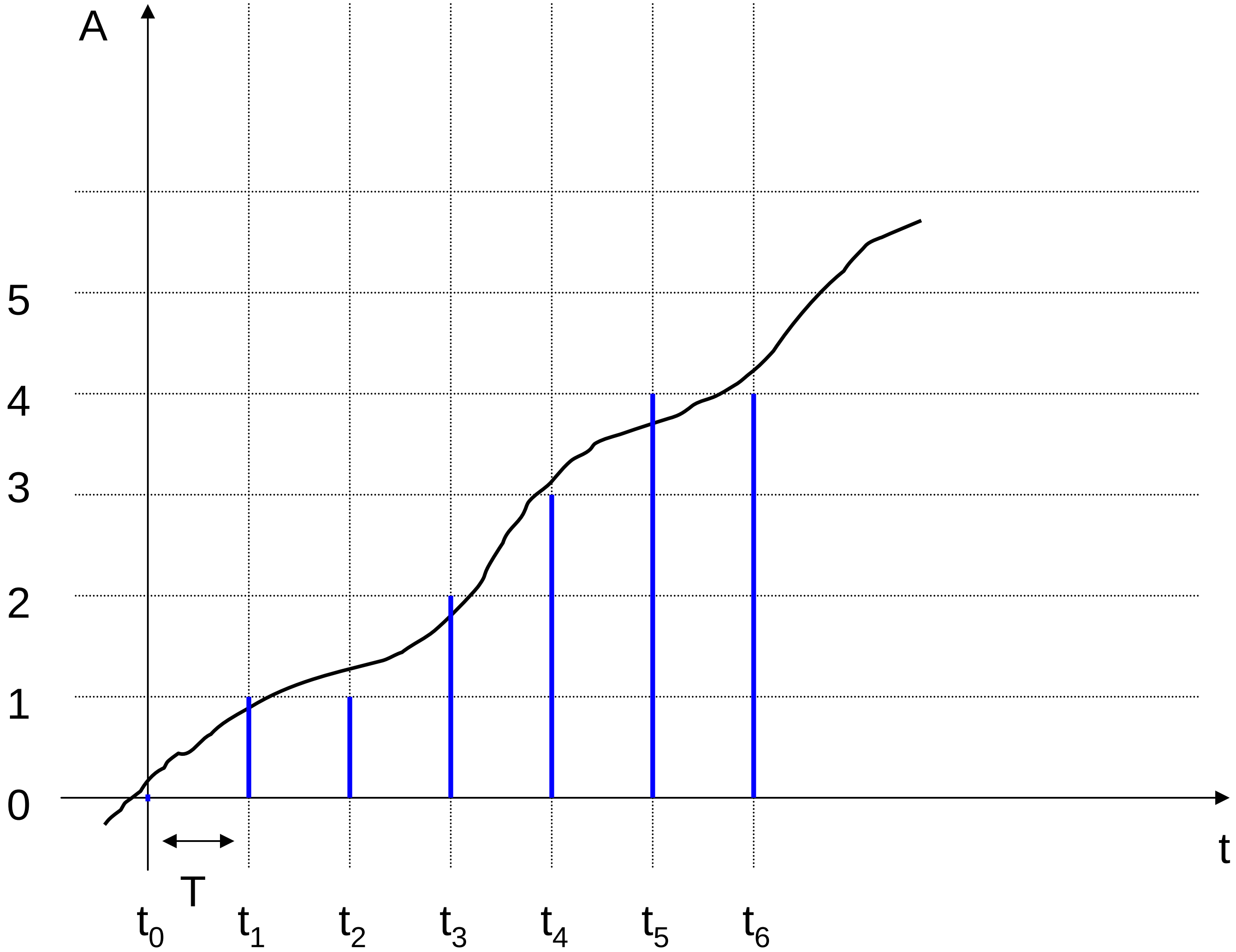
(2) # of bits per sample?!

Based on SNR formula:

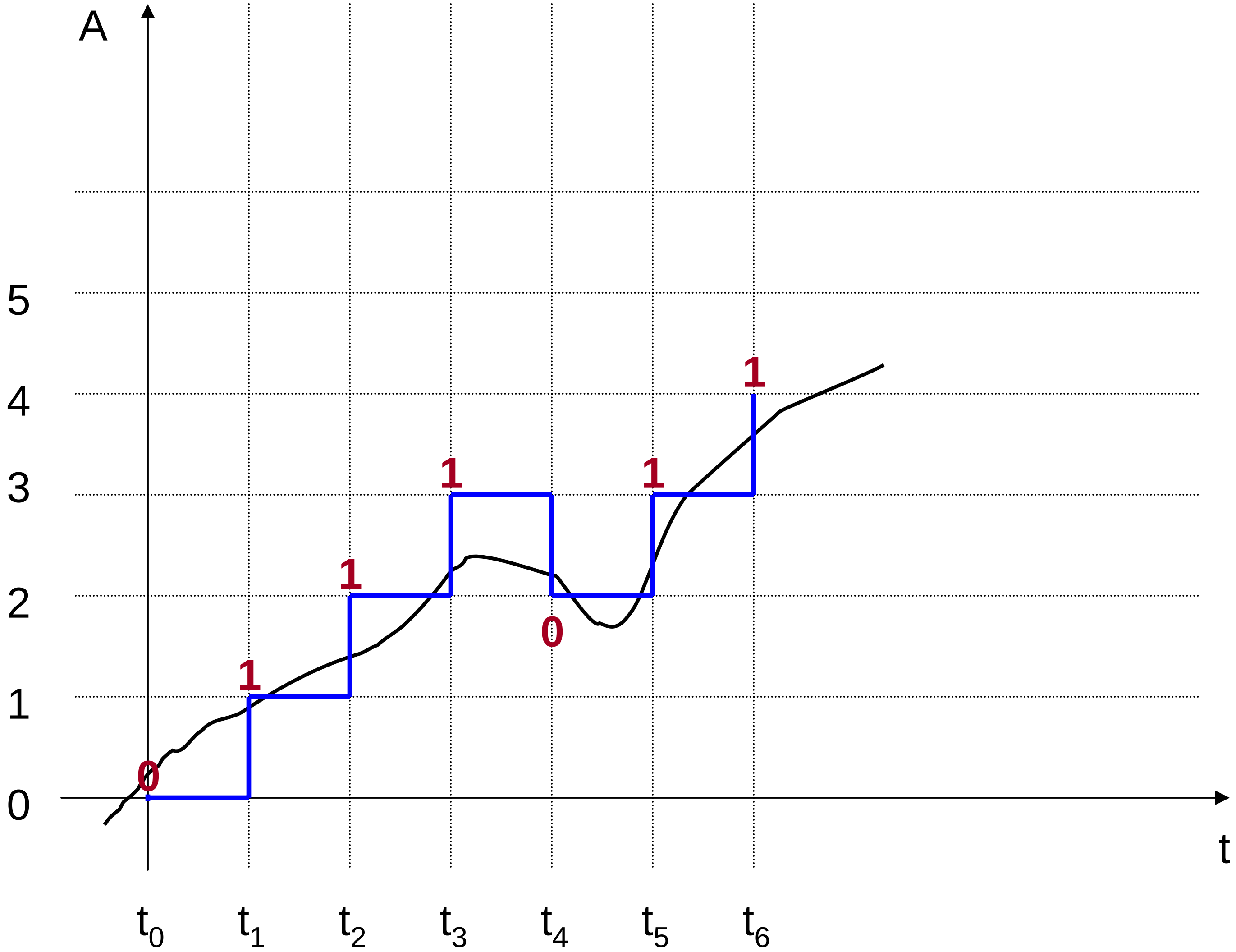
40 [dB] = 6*m + 1.76 $\Rightarrow$ # bits per sample = 7 $\Rightarrow$ # of levels = 2^7 = 127

data rate = # samples per second * # bits per sample = 56 kbps

Example [PCM]



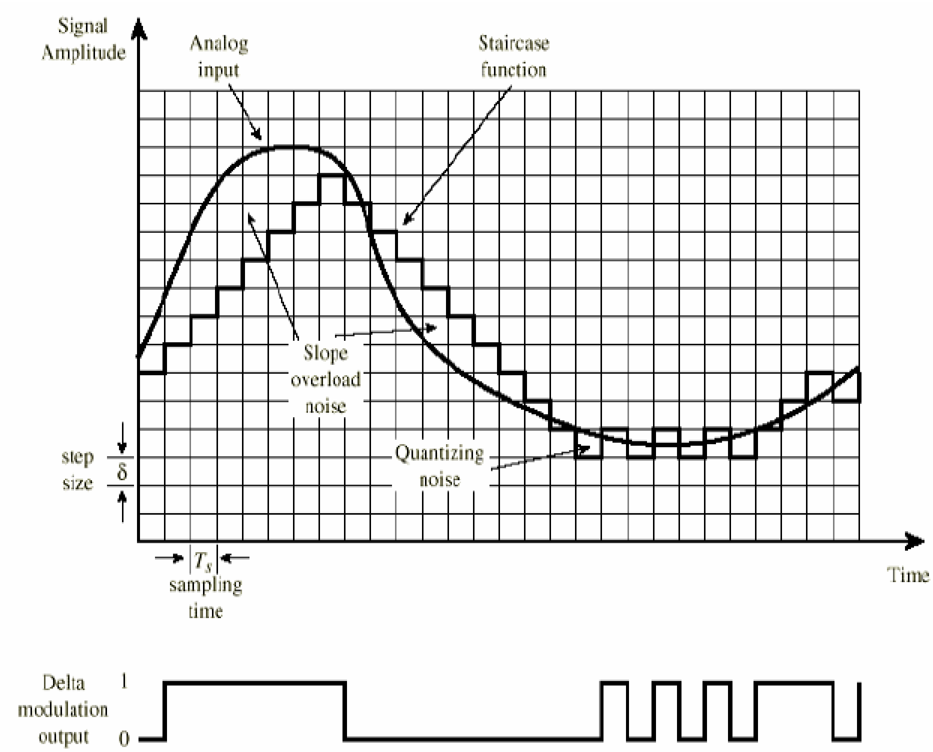
Example [Delta Modulation]



Delta Modulation

Delta-Modulation – most popular alternative to PCM

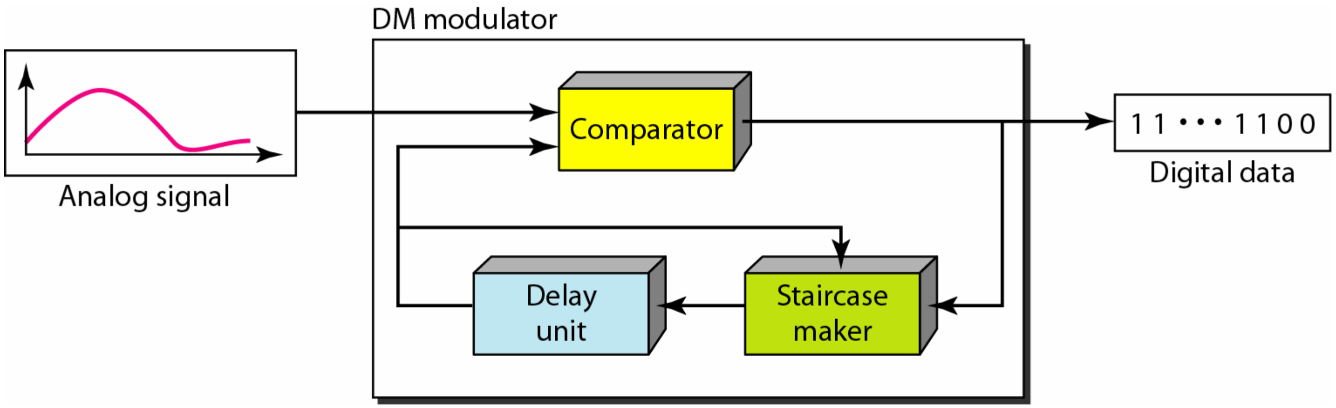
- **analog signal is approximated by staircase function**
- **only a single binary digit is required for each sample !!!**

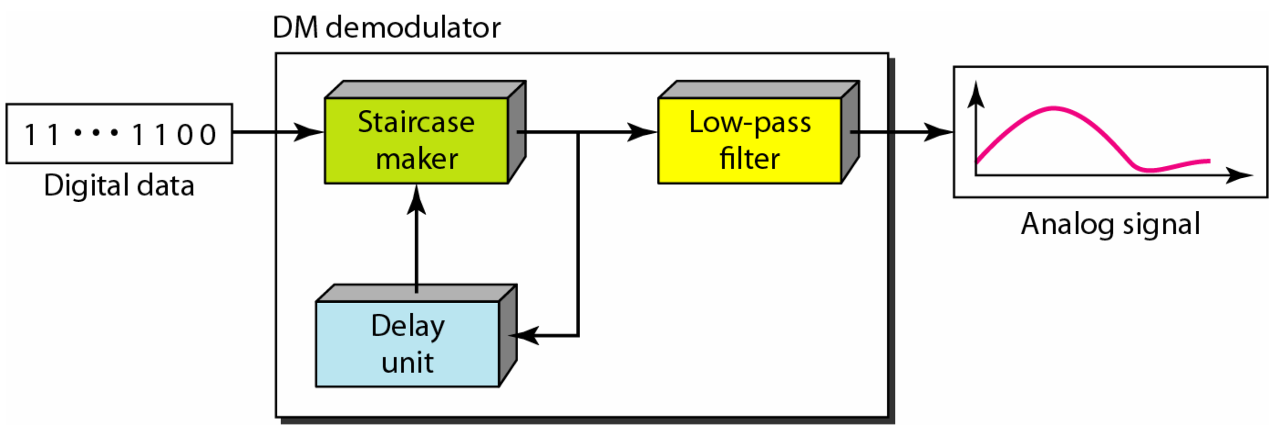


- **at each sampling time (kT), the function moves up or down a constant amount δ (step size) – the staircase function attempts to track the original waveform as closely as possible**
- **at each sampling time, the analog input is compared to the most recent value of the approximating staircase function**
- **binary-1 is generated if the function goes up, binary-0 otherwise**

Delta Modulation (cont.)

Example **[Delta modulation / demodulation]**





Delta Modulation (cont.)

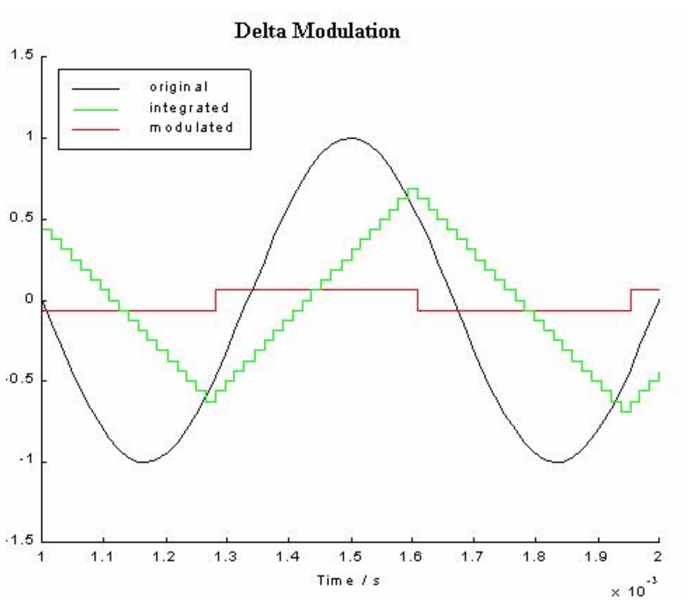
Delta Modulation Parameters

(1) step size (δ) – should not be too small, nor too large

- **small δ + signal changes rapidly ⇒ underestimation**
- **large δ + signal changes slowly ⇒ overestimation**

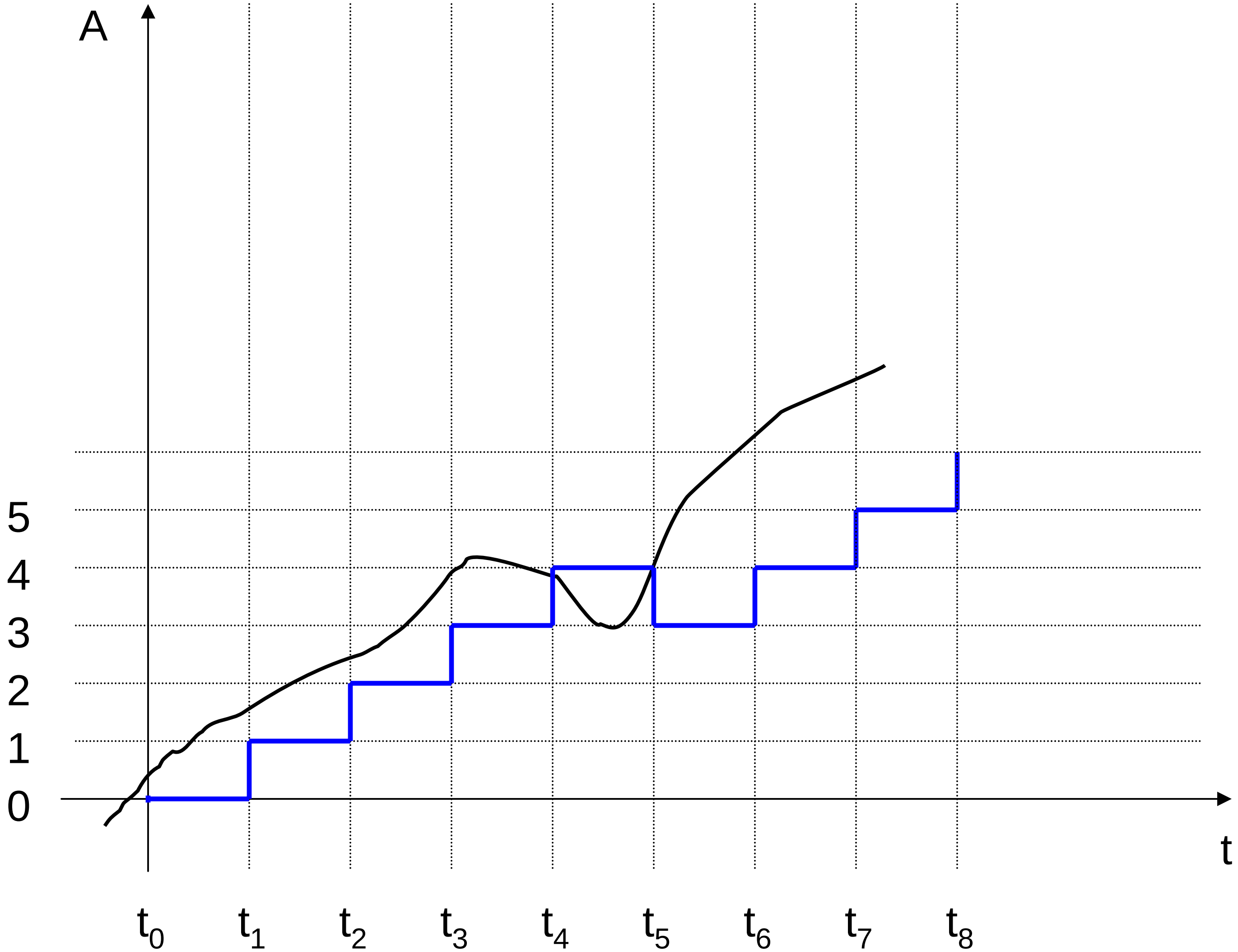
(2) sampling time (T)

- **smaller T increase overall accuracy**
- **but, small T increases output data rate, i.e. # of bps**

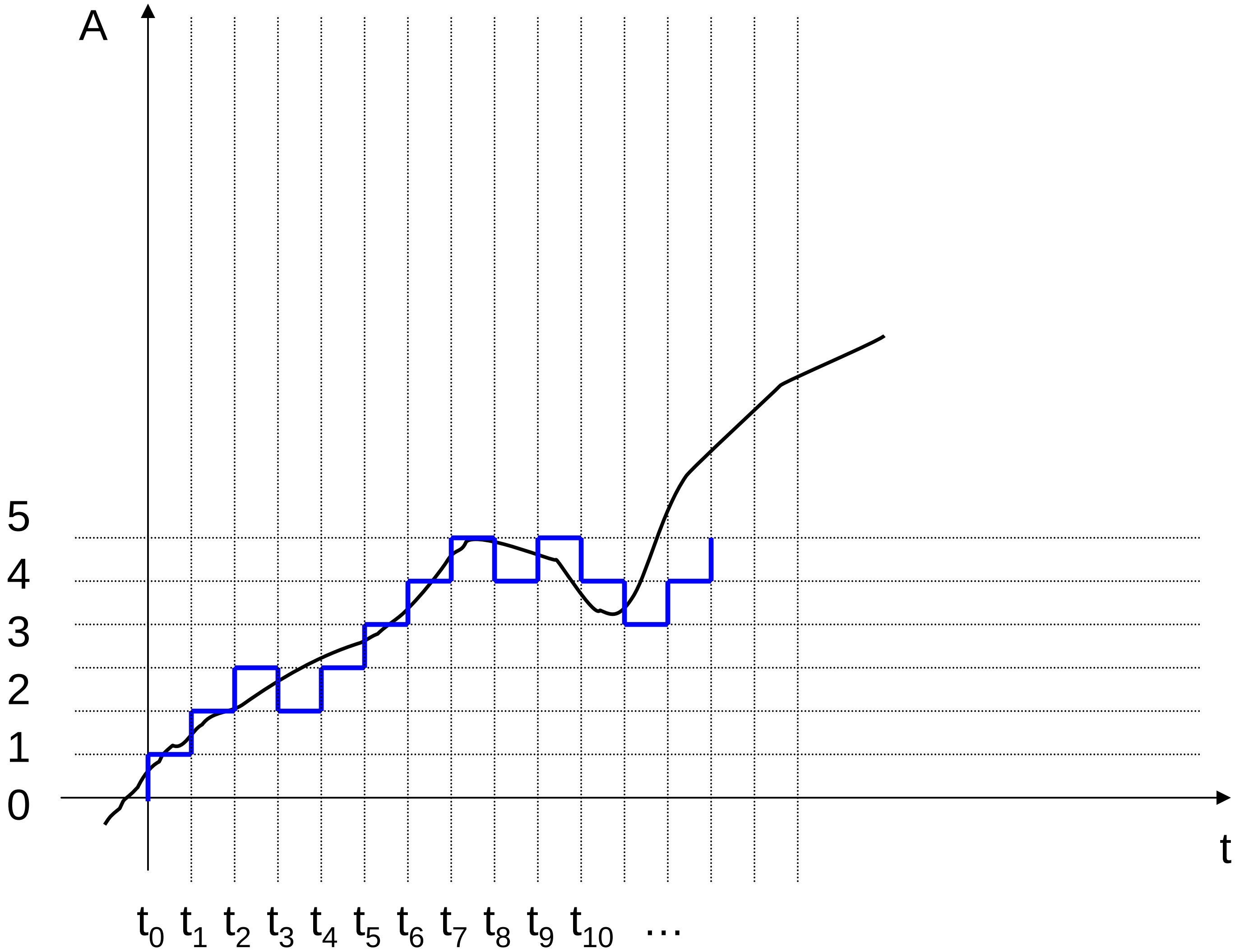


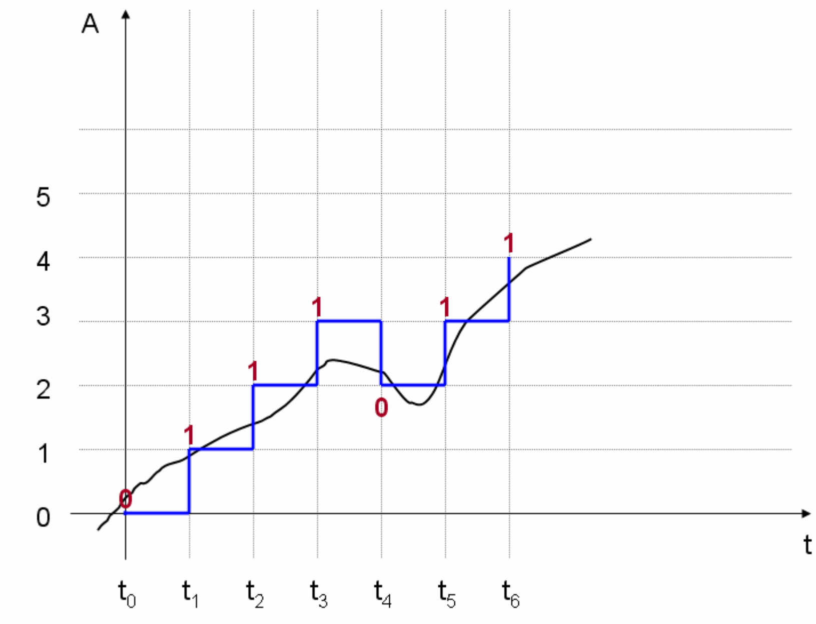
Delta-modulation rule: smaller δ ⇒ smaller T, larger δ ⇒ larger T .

Example [Delta Modulation: δ step reduced 50%, T remains the same]

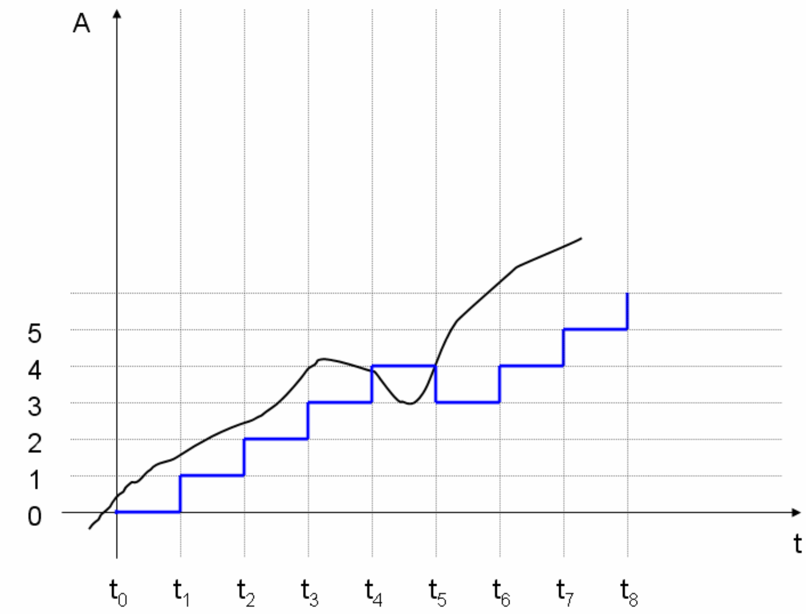


Example **[Delta Modulation: both δ-step and T reduced 50%]**

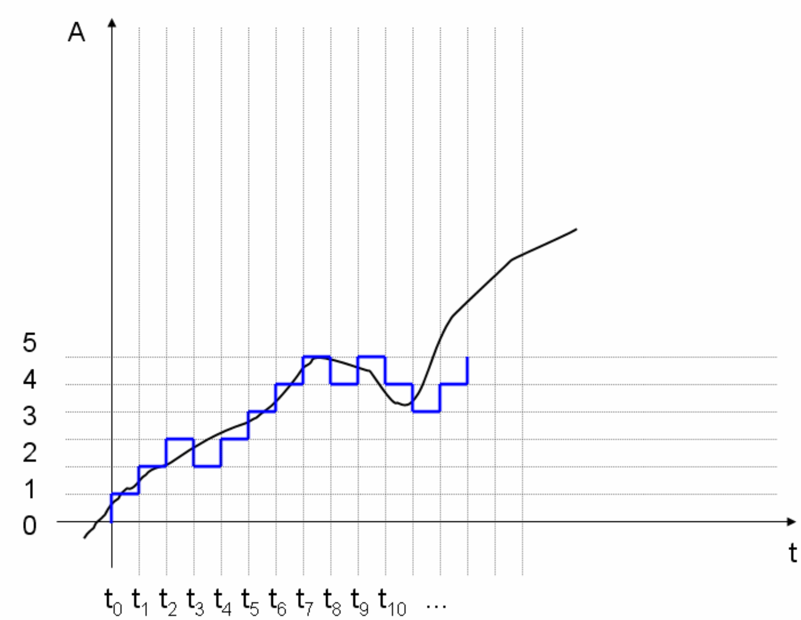




A
5
4
3
2
1
0
t
0
1
1
1
0
1
1
t_0 t_1 t_2 t_3 t_4 t_5 t_6



A
5
4
3
2
1
0
t
t_0 t_1 t_2 t_3 t_4 t_5 t_6 t_7 t_8



A
5
4
3
2
1
0
t
t_0 t_1 t_2 t_3 t_4 t_5 t_6 t_7 t_8 t_9 t_{10} …