

Homework Assignment #4

Due: October 26, 4:00 p.m.

For this assignment, you do not have to prove that your answers are correct. For full marks, your answers should be as simple as possible.

1. Consider the alphabet $\left\{ \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \end{pmatrix} \right\}$. We shall use strings in this alphabet to describe two integers: one using the top row of bits and one using the bottom row. Each integer is represented in binary. For example, to represent the two integers 13 and 7 (whose binary representations are 1101 and 111), we would use the string $\begin{pmatrix} 1 \\ 0 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \end{pmatrix}$: the top row is 1101, and the bottom row is 111 (the extra 0 at the beginning of the bottom row is just padding to make the two rows the same length).

(a) Let *LESS* be the language of all strings where the integer represented in the top row is less than the integer represented by the bottom row. For example, the string $\begin{pmatrix} 1 \\ 0 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \end{pmatrix}$ is not in *LESS*, since 13 is not less than 7. Give a regular expression for the language *LESS*.

(b) Let *LESS2* be the language of all strings where the integer represented in the top row is less than double the integer represented by the bottom row. For example, the string $\begin{pmatrix} 1 \\ 0 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \end{pmatrix}$ is in *LESS2*, since $13 < 2 \cdot 7$. Give a regular expression for the language *LESS2*.

Hint: It might be easier to find the regular expression by drawing a finite automaton for *LESS2* first.

2. Let *L* be the language described by the regular expression $1^*((0 \cup 00)11^*)^*(\varepsilon \cup 0 \cup 00)$.
- (a) List five strings that are in *L*.
- (b) List five strings that are not in *L*.
- (c) Give a precise English description of the language *L*.

Optional programming assignment (do not hand in): Write a java programme that reads a description of a finite automaton (in the same format as described in optional programming assignment #2), and outputs some string that is accepted by the automaton. If no such string exists, your progamme should output “NO STRING ACCEPTED”.