### Closures of Relations

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## Closures of Relations

- The closure of a relation R with respect to property P (such as reflexivity, symmetry, or transitivity) is the relation obtained by adding the minimum number of ordered pairs to R to obtain property P
  - To find the reflexive closure add loops
  - D To find the symmetric closure add arcs in the opposite direction
  - To find the transitive closure if there is a path from a to b, add an arc from a to b

#### Reflexive Closure

Let R be a relation on A
□ Δ={(x,x)|x∈A} is called the diagonal relation on A
□ Let R be a relation on A. The reflexive closure of R, denoted r(R), is RUΔ
Q: what is the reflexive closure of R={(a,b)|a<b} on the set of integers?</li>

#### Symmetric Closure

Let R be a relation on A  $\square R^{-1}=\{(y,x)|(x,y)\in R\}$  is called the inverse of R  $\square$  The symmetric closure of R, denoted s(R), is RUR<sup>-1</sup> Q1: what is the symmetric closure of R={(a,b)|a>b} on the set of integers? Ø Q2: Does R<sub>◦</sub>R<sup>-1</sup> equal to  $\Delta$ ? (different from f<sub>◦</sub>f<sup>-1</sup>)
  $\oslash$  Q3: Does (R<sup>-1</sup>)<sup>-1</sup> equal to R?

# Reading and Notes

Recommended exercises: 8.4: 5,7,9,11,17