Permutations and Combinations

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Permutation

For any integer n>0, the number of permutations of a set with n elements is n!

- A permutation of a set of elements is an ordering of the elements.
- E.g. the set of elements {a, b, c} can be ordered in the following ways:

  abc acb cba bac bca cab

- By the product rule, there are n(n-1)(n-2)..1=n! permutations

Suppose there are 50 students in the class,

- In how many ways can the whole class stand in a line?

50!

- In how many ways can we select three students to stand in a line?

50x49x48

r-permutation

- An r-permutation is an ordering of r elements of a set of n elements, denoted by P(n,r)
- E.g. the 2-permutations of the set of elements {a, b, c} are:

  ab ac ba bc ca cb

- By the product rule, there are n(n-1)(n-2)....(n-r+1) r-permutations

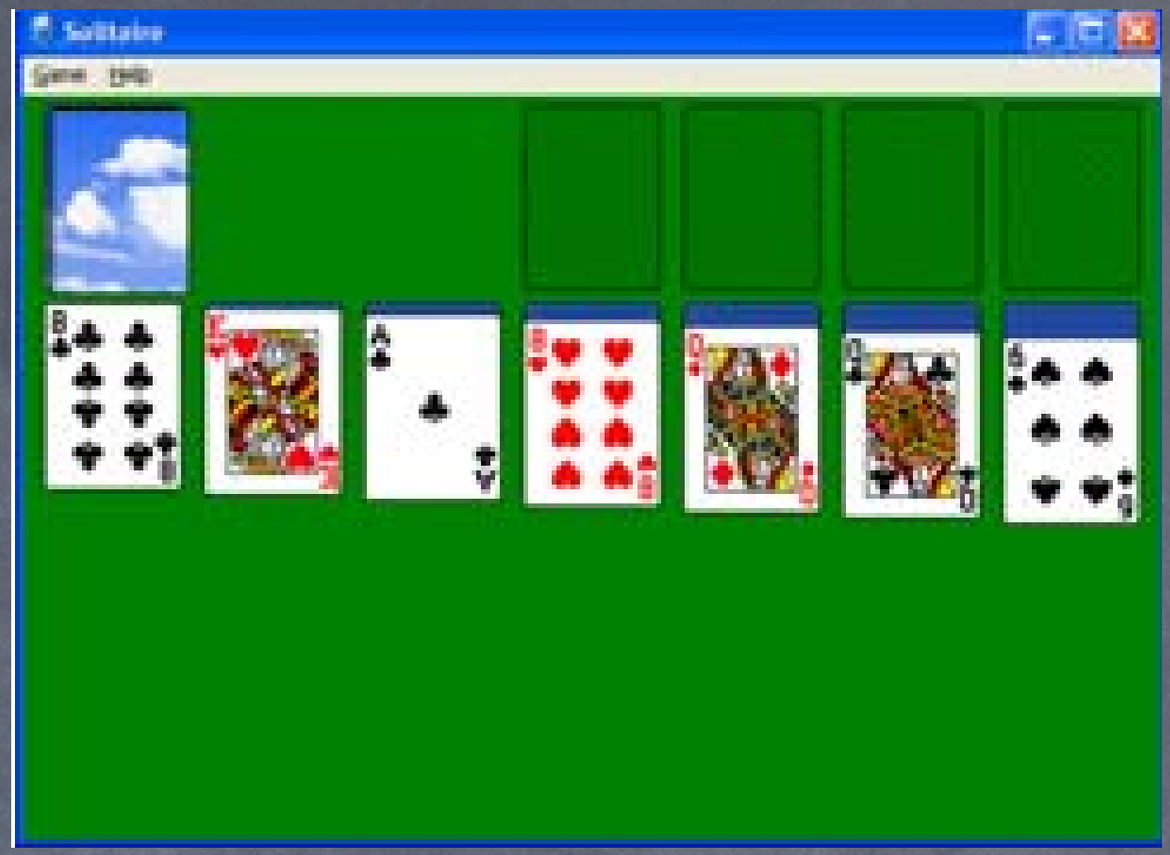
r-permutation

$$P(n,r) = n!/(n-r)! = n(n-1)(n-2)....(n-r+1)$$
$$\text{for } 0 \leq r \leq n$$

- Special cases:
  - $P(n,0) = 1$
  - $P(0,0) = 1$
  - $P(n,n) = n!$

- Recall: How many one-to-one functions are there from a set with m elements to one with n elements?

$n(n-1)\ldots(n-m+1)$ when $m \leq n$
0 when $m > n$



For the solitaire hand that show initially

- How many possible hands?

P(52,7)

- How many possible hands with no Aces?

P(48,7)

- How many possible hands with one or more Aces?

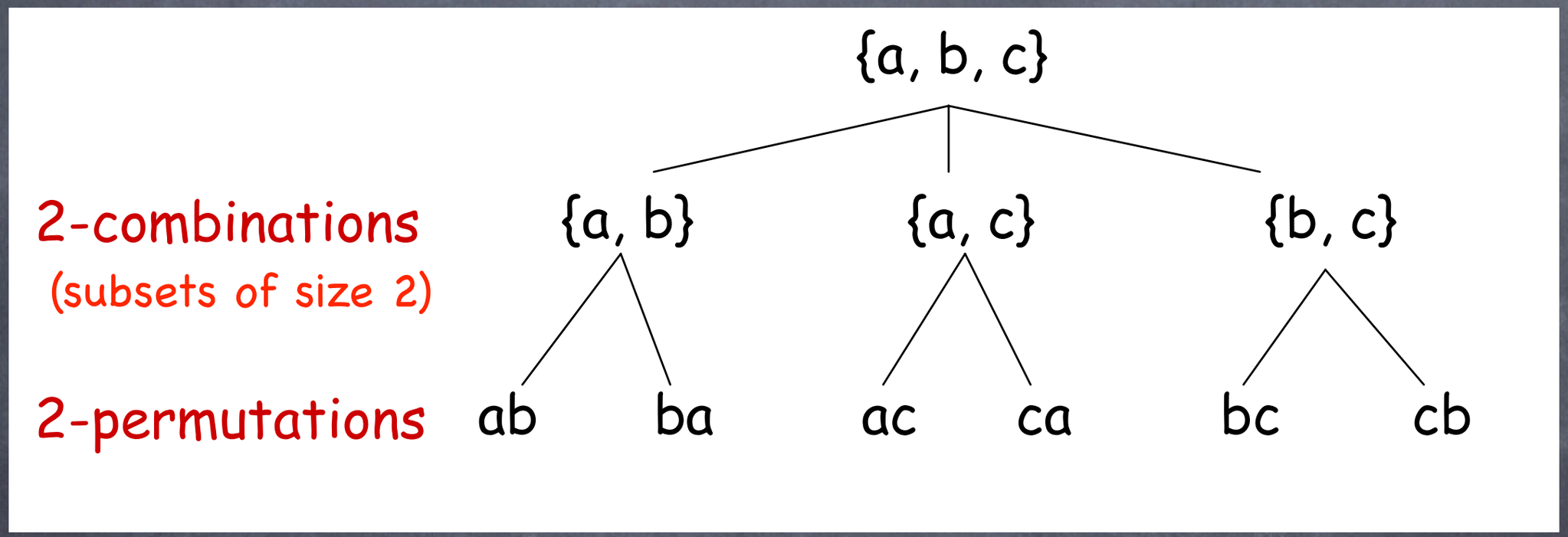
P(52,7)-P(48,7)

Combinations

- An r-combination is an unordered selection of r elements of a set of n elements, denoted by C(n,r)
- E.g. the 2-combinations of the set of elements {a, b, c} are:

{a,b} {a,c} {b,c}

Combinations & Permutations



- There are r! permutation of each subset
- There are more r-permutation than r-combinations.

Combinations

$$C(n,r) = \frac{P(n,r)}{P(r,r)} = \frac{n!}{(n-r)!r!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$$

for $0 \le r \le n$.

- Corollary: $C(n,r) = C(n,n-r)$

For a deck of 52 cards,

- How many poker hands of five cards can there be?

$$C(52,5)=2,598,960$$

- How many ways are there to select 47 cards?

$$C(52,47)=C(52,47)=2,598,960$$

Reading and Notes

- Permutations: order does matter
- Combinations: order doesn't matter
- Recommended exercises: 5.3: 3,5,7,9,12,14,17,19,21,23,28,33,43