The Basics of Counting

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Overview

Combinatorics: the study of arrangement of objects

Enumeration: the counting of objects with certain properties

Why count?

Counting Principles

Let A and B be <u>disjoint</u> sets
Sum Rule
|AUB| = |A| + |B|
Product Rule
|AxB| = |A| · |B|

Suppose there are 30 men and 20 women in a class.

How many ways are there to pick one representative from the class?
50

How many ways are there to pick two representatives, so that one is man and one is woman? Suppose you are either going to an Italian restaurant that serves 15 entrees or to a French restaurant that serves 10 entrees

How many choices of entree do you have?



Suppose you go to the French restaurant and find out that the prix fixe menu is three courses, with a choice of 4 appetizers, 10 entrees and 5 desserts

How many different meals can you have?





How many functions are there from a set with m elements to a set with n elements?

n^m

How many one-to-one functions are there from a set with m elements to one with n elements?

> n(n-1)...(n-m+1) when m≤n 0 when m>n

Principle of Inclusion-Exclusion

IF A and B are <u>not</u> disjoint sets

|AUB| = |A| + |B| - |A∩B|

Don't count objects in the intersection of two sets more than once.

What is AUBUC ?

How many positive integers less than 1000

are divisible by 7? **└**999/7 **┘** = 142 @ are divisible by both 7 and 11? **└**999/77 **┘** = 12 are divisible by 7 but not by 11? 142 - 12 = 130are divisible by either 7 or 11? 142+ _999/11 _ -12=220 are divisible by exactly one of 7 and 11? 142+ _999/11 _-12-12 = 208 @ are divisible by neither 7 nor 11? 999-220 = 779

Reading and Notes

Output Stand The difference of Sum Rule, Product Rule and Inclusion-Exclusion

Recommended exercises: 5.1: 3, 9, 19, 27, 33, 39, 43, 59