Strong Induction

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November 8 2010

Need for Strong Induction

- Review of mathematical induction
  - Basis step: P(1)
  - Inductive step: P(k)->P(k+1)
- When you cannot prove P(k+1) follows from just P(k), you may prove that P(k+1) follows from the assumption that P(j) is true for all $0<j<k+1$

Strong Induction

$$(P(1) \wedge \forall k(P(1) \wedge P(2) \wedge \ldots \wedge P(k) \rightarrow P(k+1)) \rightarrow \forall n P(n)$$

- To prove that $\forall n P(n)$, where $n \in Z^+$ and $P(n)$ is a propositional function, we complete two steps:

  i) Basis step: Verify $P(1)$ is true

  ii) Inductive step: Show $P(1) \wedge P(2) \wedge \ldots \wedge P(k) \rightarrow P(k+1)$ is true for arbitrary $k \in Z^+$

Strong Induction Variation

- A more general strong induction can handle cases where the inductive step is valid only for integers greater than a particular integer.
- To prove that P(n) is true for all integer $n \geq b$, we complete two steps:

i) Basis step: Verify P(b), P(b+1), ..., P(b+j) are true

ii) Inductive step: Show $P(1) \wedge P(2) \wedge \ldots \wedge P(k) \rightarrow P(k+1)$ is true for every integer $k \geq b+j$

Show that if n is an integer greater than 1, then n can be written as the product of primes

Proof by strong induction:

First identify P(n), P(n): n can be written as the product of primes

Basis step: P(2): 2=1x2. So, P(2) is true

Inductive step: Assume P(j) is true for $1<j\leq k$ for an arbitrary $k>1$, i.e. j can be written as the product of primes when $1<j\leq k$

Need to show P(k+1).

Case 1: k+1 is prime

Case 2: k+1 is composite

(see textbook for the details)