The Growth of Functions (2)

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Properties of Big-O

- f is $O(g)$ iff $O(f) \subseteq O(g)$
- The set $O(g)$ is closed under addition:

  If $f_1(x)$ and $f_2(x)$ are both $O(g(x))$, then $(f_1+f_2)(x)$ is $O(g(x))$
- The set $O(g)$ is closed under multiplication by a scalar a ($a \in R$):

  If f is $O(g)$ then af is $O(g)$

Properties of Big-O

- If f is $O(g)$ and g is $O(h)$ then $f=O(h)$

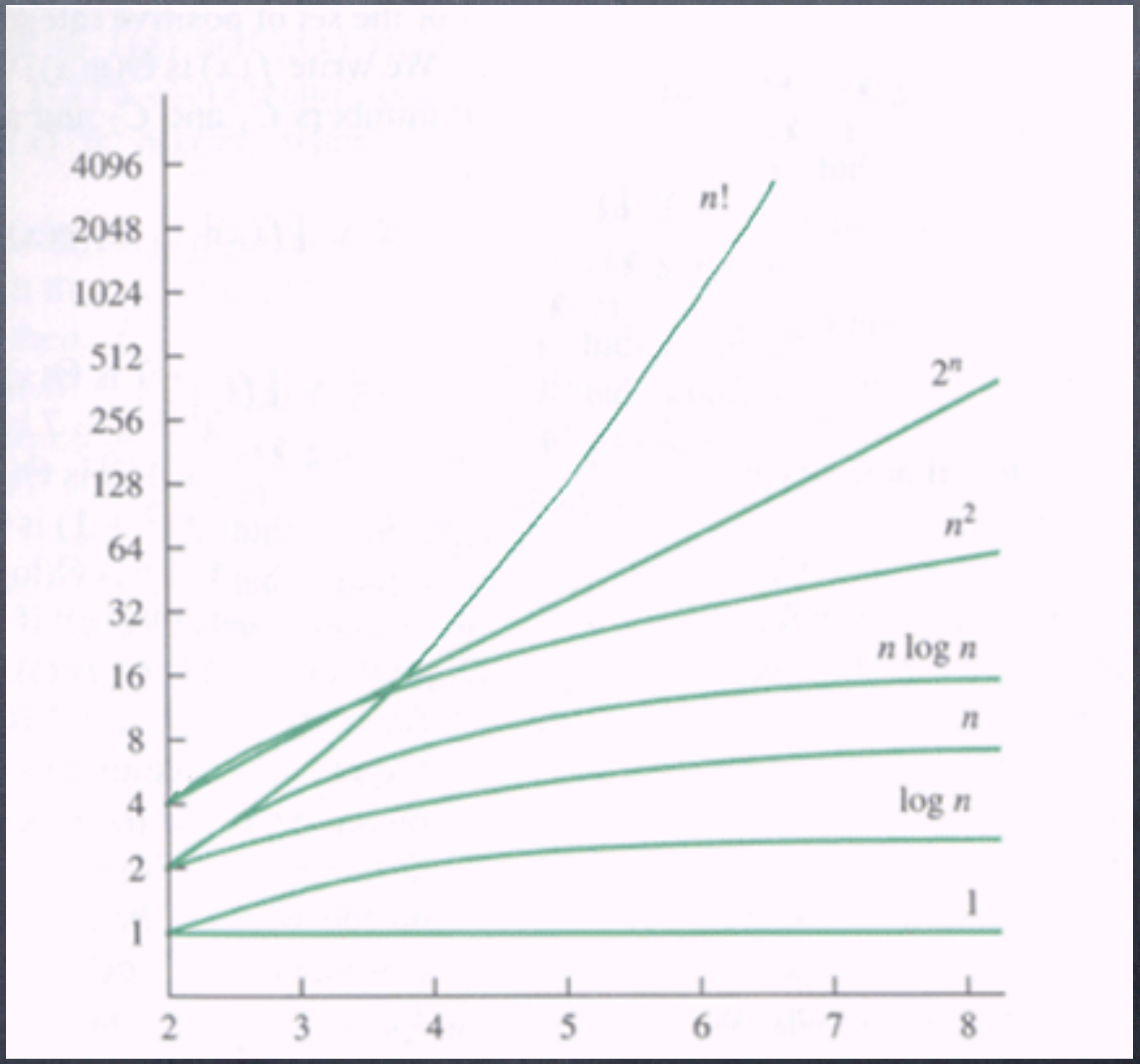
  $O(f) \subseteq O(g) \subseteq O(h)$

- If f_1 is $O(g_1)$ and f_2 is $O(g_2)$ then
  - $f_1 f_2$ is $O(g_1 g_2)$
  - f_1+f_2 is $O(\max\{g_1,g_2\})$

Important Complexity Classes

$O(1) \subseteq O(\log n) \subseteq O(n) \subseteq O(n\log n) \subseteq O(n^2) \subseteq O(n^j) \subseteq O(c^n) \subseteq O(n!)$

where $j>2$ and $c>1$



Some crucial facts

- Logarithmic << Polynomial

  $\log^{1000} n \ll n^{0.001}$ for sufficiently large n

- Linear << Quadratic

  $1000n \ll 0.0001n^2$ for sufficiently large n

- Polynomial << Exponential

  $n^{1000} \ll 2^{0.001n}$ for sufficiently large n

Find the complexity class of the function

$$(nn!+3^{n+2}+3n^{100})(n^n+n2^n)$$

Solution: This means to simplify the expression

Throw out stuff which you know doesn't grow as fast.

Use the property that if f is O(g) then f+g is O(g)

(i) For $nn!+3^{n+2}+3n^{100}$, eliminate 3^{n+2} and $3n^{100}$ since n! grows faster than both of them

(ii) Now simplify n^n+n2^n, which grows faster?

Take the log (base 2) of both (since the log is an increasing function whatever conclusion we draw about the logs will also apply to the original functions)

Compare nlogn and logn+n, nlogn grows faster so we keep n^n.

The complexity class is $O(nn!n^n)$

Big-Omega

- Assume $f: Z \rightarrow R$ and $g: Z \rightarrow R$.

$f(x)$ is $\Omega(g(x))$ iff $\exists$ positive constants C and k such that

$$\forall x > k \; |f(x)| \geq C|g(x)|$$

Big-O and Big-Omega

- Big-O provides upper bound for functions
- Big-Omega provides lower bound for functions

Big-Theta

- Assume $f:Z \rightarrow R$ and $g: Z \rightarrow R$.

  $f(x)$ is $\Theta(g(x))$ iff $f(x)=O(g(x))$ and $f(x)=\Omega(g(x))$

- Big-Theta Θ provides both upper and lower bounds for functions

Readings and Notes

- Understand the definition and application of Big-O, Big-Omega, Big-Theta notation
- Practice determining the big-O function for a function
- Recommended exercises: 3.2: 1,19,23