Sequences & Summations (1)

Jing Yang
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Sequences

- A sequence is an ordered list, possibly infinite, of elements,

  notated by $\{a_1, a_2, a_3 \ldots\}$ or $\{a_i\}_{i=0}^{k}$

  where k is the upper limit (usually ∞)

sequences vs sets?

- A sequence is a function from a subset of the Z (usually {0,1,2,...}) to another set

- a_n is the image of the the integer n. We call a_n a term of the sequence, and n is its index or subscript

Arithmetic Progressions

- An arithmetic progression is a sequence of the form

$$a,\ a+d,\ a+2d,\ a+3d,\ \ldots,\ a+(n-1)d,\ldots$$

Initial term a

Common difference d

- E.g.
  - {-1, 3, 7, 11, ...}
  - {7,4,1,-2, ...}

Geometric Progressions

- An geometric progression is a sequence of the form

$$a,\ ar,\ ar^2,\ ar^3,\ \ldots,\ r^n,\ldots$$

Initial term a

Common ratio r

- E.g.
  - {1,-1,1,-1,1, ...}
  - {2,10,50,250,1250, ...}

Useful Sequences

- $\{n^2\}$: 1, 4, 9, 16, 25, ...
- $\{n^3\}$: 1, 8, 27, 64, 125, ...
- $\{n^4\}$: 1, 16, 81, 256, 343, ...
- $\{2^n\}$: 2, 4, 8, 16, 32, ...
- $\{3^n\}$: 3, 9, 27, 81, 243, ...
- $\{n!\}$: 1, 2, 6, 24, 120, ...

Summations

- Given a sequence $\{a_i\}$ the summation notation for its terms a_m, a_{m+1},..., a_n

$$\sum_{i=m}^{n} a_i, \quad \sum_{i=m}^{n} a_i, \quad or \sum_{m<i<n} a_i$$

represent $a_m+a_{m+1}+...+a_n$

- E.g.
  - $r^0+r^1+r^2+...+r^n = \sum_{i=0}^{n} r^i$
  - $1+1/2+1/3+1/4+... = \sum_{i=1}^{\infty} \frac{1}{i}$

Summation of Arithmetic Progression

- Given a arithmetic progression a, a+d, a+2d, a +3d,. . . its summation is

$$\sum_{i=0}^{n} a + id = \frac{(2a + nd)(n + 1)}{2}$$

- How to prove it?
- You should also be able to determine the sum if the index starts at k and/or ends at n-1, n+1, etc.

Summation of Geometric Progression

- Given a geometric progression a, ar, ar^2, ar^3,. . . its summation is

$$\sum_{i=0}^{n} ar^i = \frac{ar^{n+1} - a}{r - 1} \quad if\ r \neq 1$$

(you can figure out what it is if r=1)

- How to prove it?
- You should also be able to determine the sum if the index starts at k and/or ends at n-1, n+1, etc.