Set Operations

Jing Yang

October 4, 2010

Review

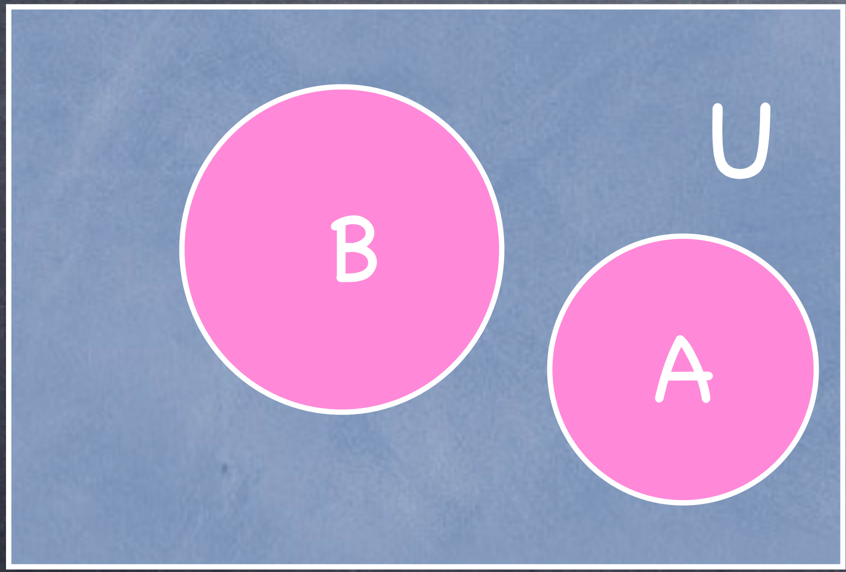
- Sets: definition, representation
- Set membership, subsets, proper subsets, power set, Cartesian product
- Today: set operations

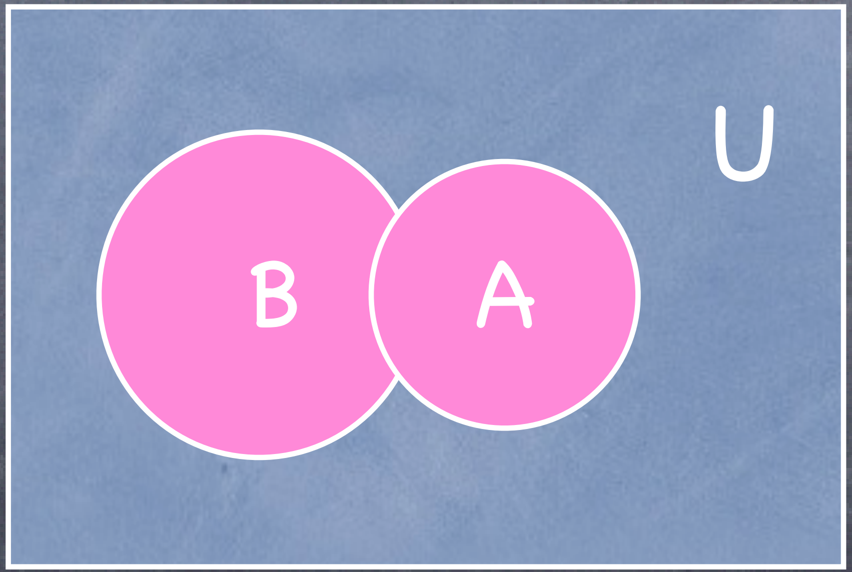
Set Operations

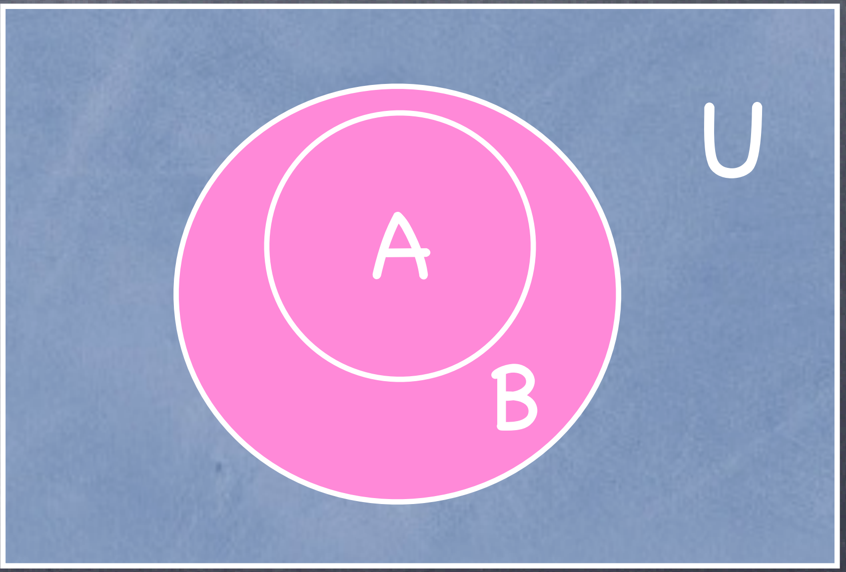
- Union: $A \cup B = \{x | (x \in A) \vee (x \in B)\}$
- Intersection: $A \cap B = \{x | (x \in A) \wedge (x \in B)\}$
  - Disjoint sets: A, B are disjoint iff $A \cap B = \emptyset$
- Difference: $A - B = \{x | (x \in A) \wedge (x \notin B)\}$
- Complement: A^c or $\bar{A} = \{x | x \notin A\} = U - A$

Union A∪B

- Union: $A \cup B = \{x \mid (x \in A) \vee (x \in B)\}$

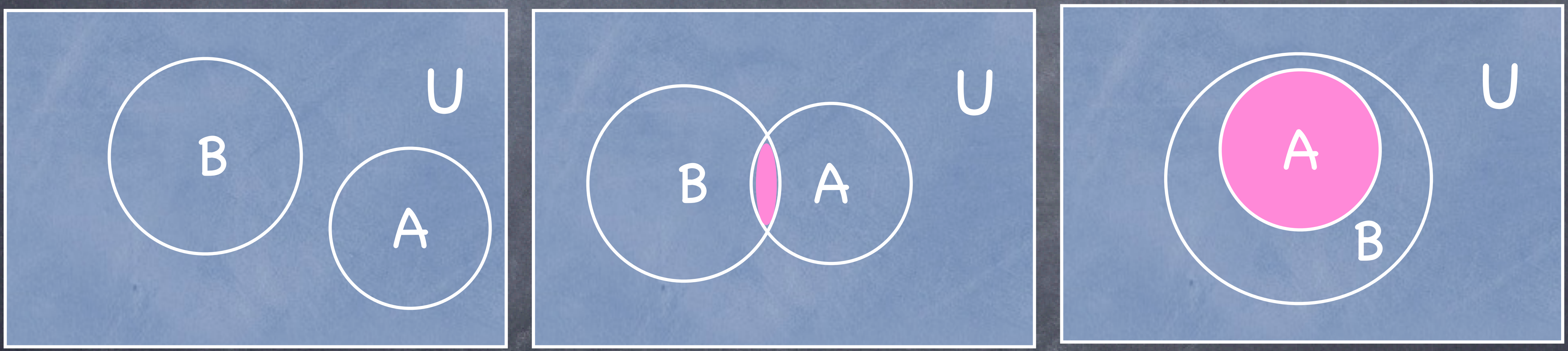






Intersection A∩B

- **Intersection:** $A \cap B = \{x \mid (x \in A) \wedge (x \in B)\}$

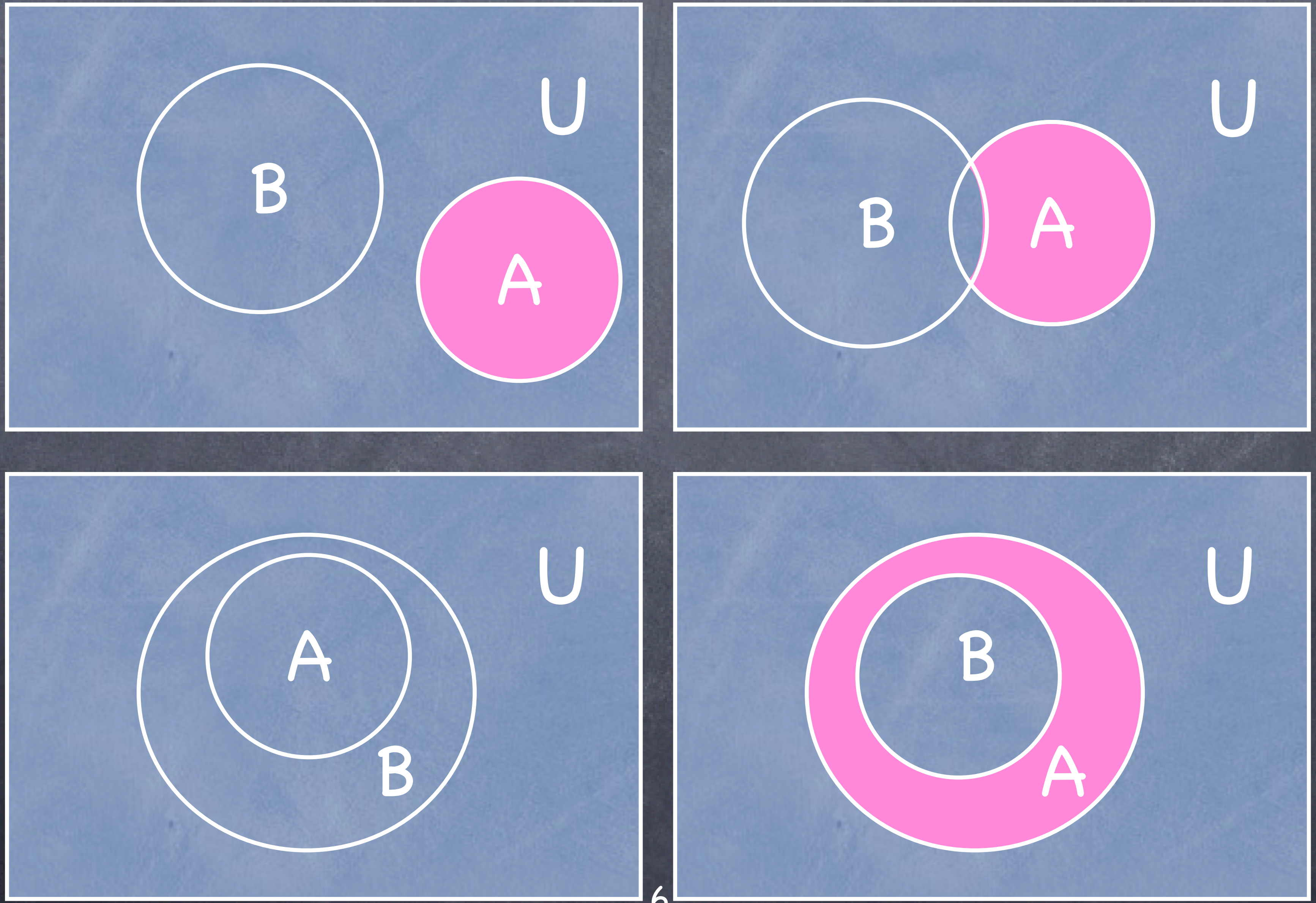


Disjoint sets

A, B are disjoint iff $A \cap B = \emptyset$

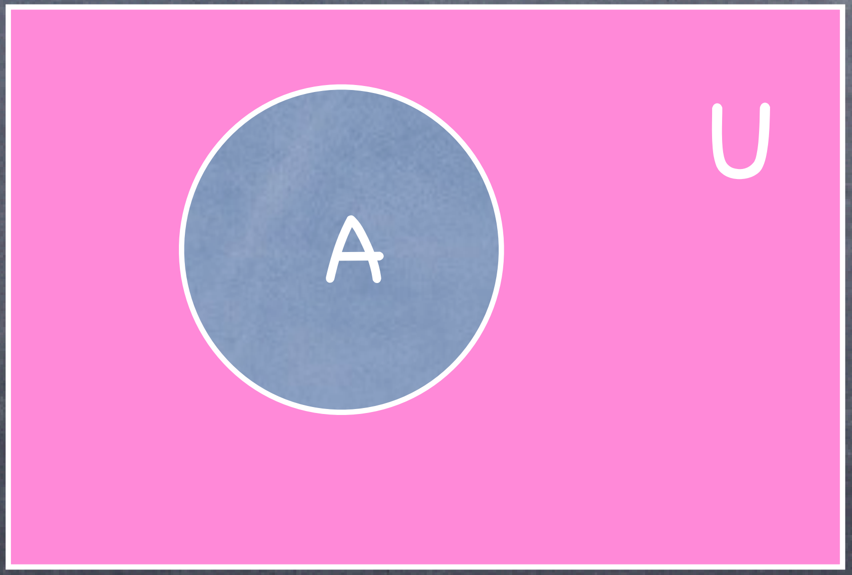
Difference A-B

- **Difference:** A-B = $\{x \mid (x \in A) \land (x \notin B)\}$



Complement A^c or $\bar{A}$

- **Complement:** A^C or $\bar{A} = \{x \mid x \notin A\} = U - A$



Examples

For $U = \{0,1,2,3,4,5,6,7,8,9,10\}$,

If $A = \{1,2,3,4,5\}$, $B = \{4,5,6,7,8\}$, then:

- $A \cup B = \{1,2,3,4,5,6,7,8\}$
- $A \cap B = \{4,5\}$
- $A - B = \{1,2,3\}$
- $B - A = \{6,7,8\}$
- $\bar{A} = \{6,7,8,9,10\}$

Set Identities

- Set identities correspond to logical equivalences
- How to prove the set identities?
  - Show each set is a subset of the other
  - Use membership tables

Example: $\overline{A \cup B} = \overline{A} \cap \overline{B}$

- Proof by showing: $\forall x(x \in \overline{A \cup B} \leftrightarrow x \in \overline{A} \cap \overline{B})$
- Let x be arbitrary, then we can treat the predicates as propositions

Example: $\overline{A \cup B} = \overline{A} \cap \overline{B}$

$x \in \overline{A \cup B}$	
$\equiv x \notin A \cup B$	Def. of complement
$\equiv \neg(x \in A \cup B)$	Def. of $\notin$
$\equiv \neg(x \in A \vee x \in B)$	Def. of $\cup$
$\equiv \neg(x \in A) \wedge \neg(x \in B)$	De Morgan's Laws
$\equiv x \notin A \wedge x \notin B$	Def. of $\notin$
$\equiv x \in \overline{A} \wedge x \in \overline{B}$	Def. of complement
$\equiv x \in \overline{A} \cap \overline{B}$	Def. of $\cap$

Example: $\overline{A \cup B} = \overline{A} \cap \overline{B}$

- Since:
  - x was arbitrary
  - By using only logically equivalent assertions and definitions we showed

$$x \in \overline{A \cup B} \leftrightarrow x \in \overline{A} \cap \overline{B}$$

  is a tautology
- So we can claim:

$$\forall x (x \in \overline{A \cup B} \leftrightarrow x \in \overline{A} \cap \overline{B})$$

Example: $\overline{A \cup B} = \overline{A} \cap \overline{B}$

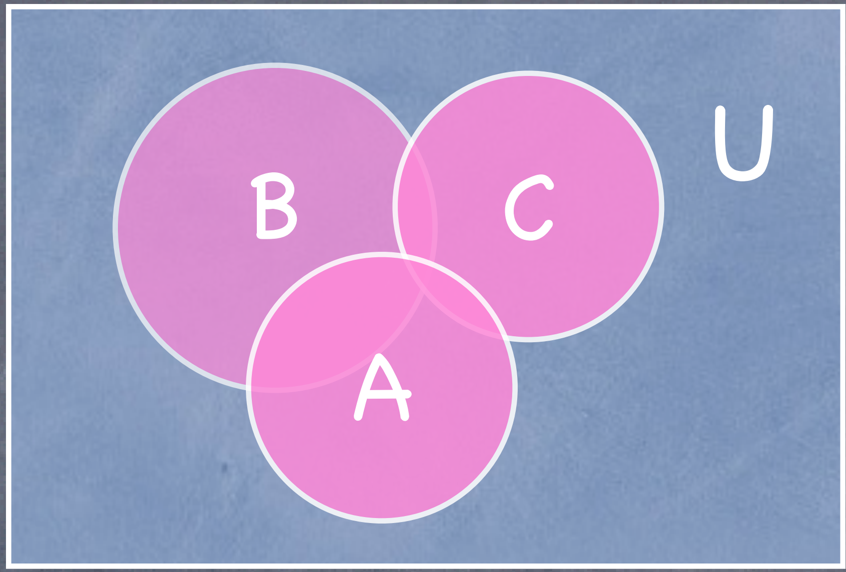
- Proof by using a membership table

A	B	A∪B	$\overline{A \cup B}$	$\overline{A}$	$\overline{B}$	$\overline{A} \cap \overline{B}$
1	1	1	0	0	0	0
1	0	1	0	0	1	0
0	1	1	0	1	0	0
0	0	0	1	1	1	1

Set Identities

- Important set identities (Page 124)
  - Associative Laws, Distributive Laws, De Morgan's Laws
  - Similarity of the laws in P124 and P24

Generalized Unions

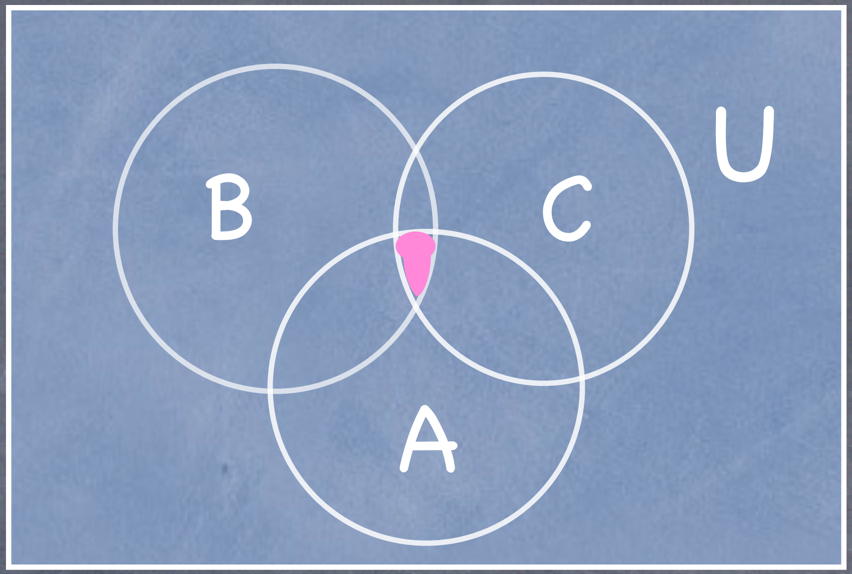


- A∪B∪C

- Let A1, A2, ..., An be an indexed collection of sets, we have:

$$\bigcup_{i=1}^{n} A_i = A_1 \cup A_2 \cup \ldots \cup A_n$$

Generalized Intersections



- $A \cap B \cap C$
- Let A_1, A_2, ..., A_n be an indexed collection of sets, we have:

$$\bigcap_{i=1}^{n} A_i = A_1 \cap A_2 \cap \ldots \cap A_n$$

Example

- Let $A_i=\{i,i+1,i+2,...\}$, then:

$$\bigcup_{i=1}^{n} A_i = \{1, 2, 3, ...\}$$

$$\bigcap_{i=1}^{n} A_i = \{n, n+1, n+2, ...\}$$

Reading and Notes

- Understand the relationship between set operations and logic operations
- Practice proving set identities
- Recommended exercises: 3,5,19,25