Sets

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Sets

- Unordered collection of distinct objects (called the elements, or members, of the set)
- Elements could be:
  - Positive integers
  - Sides of a coin
  - Students enrolled in 1019A
  - Sets

Set Membership

- $a \in A$: a is an element of the set A
- $a \notin A$: a is not an element of the set A
- Example:
  - V: {a,e,i,o,u} -- $a \in V$, $b \notin V$
  - T: {1, 2, 3, 4, …, 99} -- $55 \in T$, $100 \notin T$
  - S: {a, 2, {a}} -- $a \in S$, $\{a\} \in S$, $\{\{a\}\} \notin S$

Describing Sets

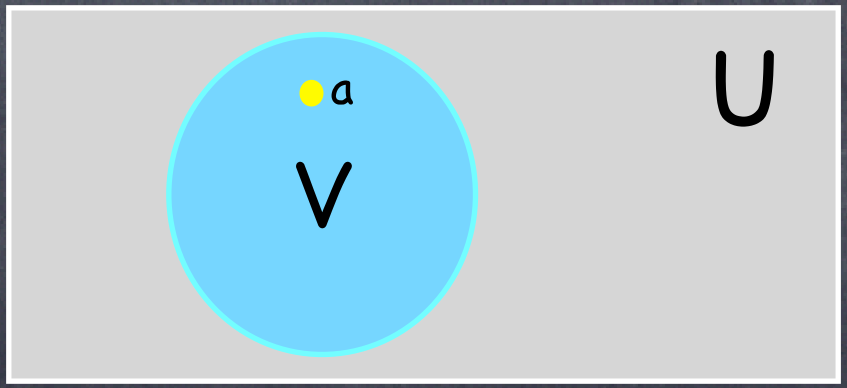
- Brace notation: List the members in { }
- Set builder notation (specification by predicates):

$$S = \{x \mid P(x)\}$$

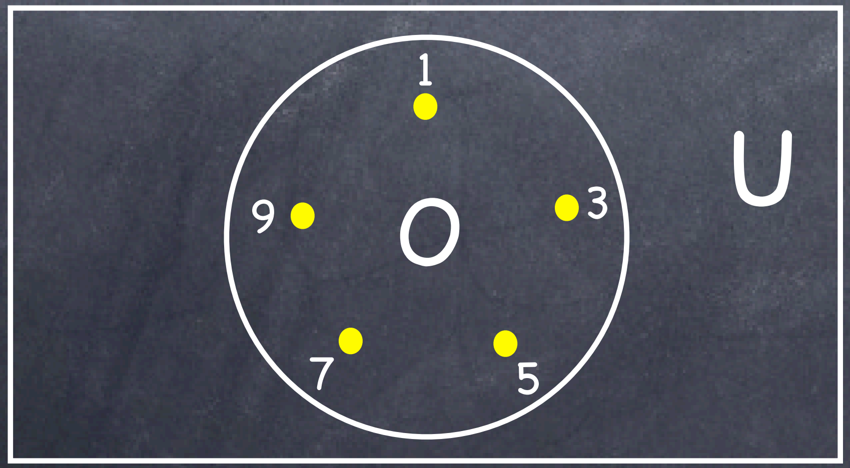
  - S contains all the elements which make P(x) true
  - Characterize all elements in the set by stating properties they must have
  - E.g. O = {x | x is an odd positive integer less than 10}

Describing Sets - 2

- Venn Diagrams: Rectangle, circles, points
  - Universal set U contains all the objects under consideration
  - Often used to show relationships between sets



Set Examples

- O = {1, 3, 5, 7, 9}
- O = {x | x is an odd positive integer less than 10}
  - O = {x | x is odd and x>0 and x<10}
- 

Important Sets

- Set of natural numbers: N = {0,1,2,3,...}
- Set of integers: Z = {...,-2,-1,0,1,2,...}
- Set of positive integers: Z^+ = {1,2,3,...}
- Set of rational numbers: Q = {p/q | p∈Z, q∈Z, and q≠0}
- Set of real numbers: R

Size of Sets

Let S be a set

- Cardinality $|S|$: number of (distinct) elements
- Finite set: cardinality is some finite integer n
- Infinite set: a set that is not finite

Special sets

- Empty set: $\varnothing$ or { }
- Singleton set: A set with one element

Equivalence of Sets

- Two sets A and B are equal iff they have the same elements, i.e. $A=B$ iff $\forall x(x\in A\leftrightarrow x\in B)$.
- E.G.
  - $\{1,2,3\} = \{3,1,2\}$
  - $\{1,1,1\} = \{1\}$
  - $\{\emptyset,\emptyset\} = \{\emptyset\} \neq \emptyset$
  - $Z \neq N$

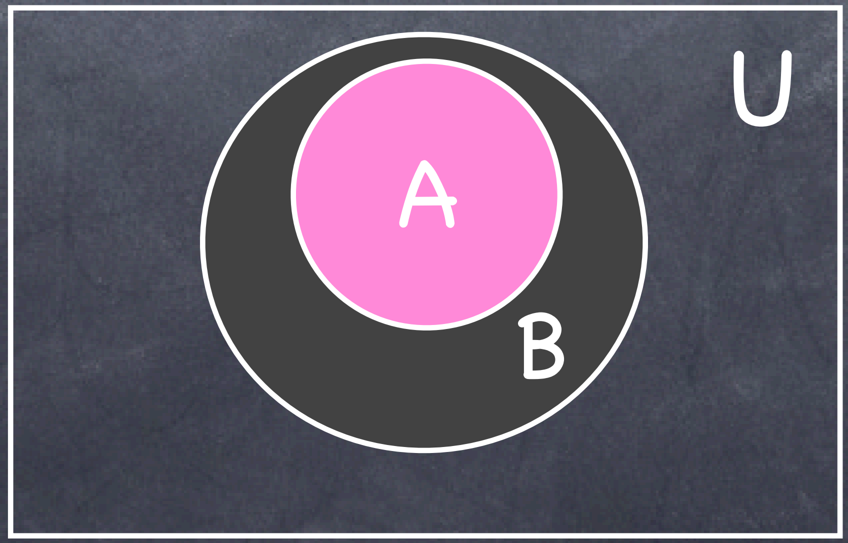
Subsets

- A⊆B: Every element of A is also an element of B.
  i.e. $\forall x(x \in A \rightarrow x \in B)$

- Proper subset A⊂B: A⊆B but A≠B

  $\forall x(x \in A \rightarrow x \in B) \wedge \exists x(x \in B \wedge x \notin A)$

- A=B if and only if A⊆B and B⊆A



Special Subsets

- Theorem: For every set S
  - $S \subseteq S$
  - $\emptyset \subseteq S$
  - $S \subseteq U$
- Proof in the textbook

Power Set

- Power Set P(S): set of all subsets of S
  - P(S) includes S, ∅
  - If |S|=n then $|P(S)|=2^n$
- E.G.
  - If A = {a,b}, then P(A)={∅,{a},{b},{a,b}}
  - Tricky question: What is P(∅) and P({∅})?

Cartesian Products

- The Cartesian product of A with B, denoted AXB is the set of ordered pairs {<a,b> | a∈A ∧ b∈B}
  - Notation: $\times_{i=1}^{n} A_i = \{< a_1, a_2, ..., a_n > | a_i \in A_i\}$
- Example: A = {a,b}, B={1,2,3}

AxB = {<a,1>,<a,2>,<a,3>,<b,1>,<b,2>,<b,3>}

BxA = {<1,a>,<1,b>,<2,a>,<2,b>,<3,a>,<3,b>}

Cartesian Products - 2

- If $|A| = m$ and $|B| = n$, then $|A \times B| = mn$.
- The cartesian product of anything with $\emptyset$ is $\emptyset$. Why?

Readings and Notes

- Set is one of the basic discrete data structures.
- Understand the concept of sets, set membership, subset, cardinality, powerset, cartesian product of sets.
- Recommended exercises: 1,5,7,13,19,25,35